Radiation Energy Loss of Relativistic Charge in Matter

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Abstract

Radiation produced by relativistic charge in matter is considered in terms of transverse energy loss. Optical, ultraviolet and X-ray Cherenkov radiation, prompt bremsstrahlung, the Tamm problem, normal and anomalous Doppler effects, ionisation energy losses, and transition radiation in complex radiators are discussed. Geant4 simulations are in satisfactory agreement with experimental data.

1 Introduction. The Poynting's theorem in absorbing medium

- Relativistic particle energy loss in a medium can be subdivided to Cherenkov (CL, $\perp$) and Bohr (BL, $\|$) losses, i.e. the works done by the particle against the transverse and longitudinal (relative to the wave vector) components of its electric field in the current particle position, respectively.
- In terms of quantum approach CL and BL are responsible for the generation of the transverse (photons in medium) and longitudinal (plasmons, delta-electrons) medium excitations along the particle trajectory, respectively.
- The aim of the present report is to discuss the concept of CL and its implementation in the framework of Geant4 [1] with illustrations showing the comparison between the experimental data and simulation results.

Let us consider a relativistic charged particle with the charge e moving in an absorbing medium with the complex dielectric permittivity $\epsilon = \epsilon_1 + i\epsilon_2$. The particle energy loss Δ will be calculated in the framework the condition when the particle velocity $\mathbf{v}$ is supposed to be constant (so called, adiabatic approximation). The Poynting's theorem can be derived directly from the Maxwell's equations and after integration over a large volume V surrounding the particle trajectory results in [2]:

$$-e\mathbf{v}\mathbf{E}(\mathbf{v}t, t) = \frac{d\bar{\Delta}}{dt} = \frac{1}{4\pi}\int_V \left\{\mathbf{E}\frac{\partial \mathbf{D}}{\partial t} + \mathbf{H}\frac{\partial \mathbf{B}}{\partial t}\right\} d\mathbf{r} + \frac{c}{4\pi}\oint_S \mathbf{E}\times\mathbf{H}d\mathbf{s}, \qquad (1)$$

where $\mathbf{E}$ and $\mathbf{D}$ are the electric field and induction, $\mathbf{H}$ and $\mathbf{B}$ are the magnetic field and induction, respectively; $\bar{\Delta}$ is the mean energy loss of the particle, c is the speed of light in vacuum, $d\mathbf{s}$ is the element of the surface S surrounding V.

Let a relativistic charged particle with the charge e move along arbitrary trajectory $\mathbf{r}(t)$ in an absorbing medium with the complex dielectric permittivity $\epsilon = \epsilon_1 + i\epsilon_2$ and magnetic permeability $\mu = \mu_1 + i\mu_2$. The mean number of photons $\bar{N}_\perp$ emitted into unit solid angle Ω, per unit energy $\hbar\omega$, in unit time t reads (at a given frequency: $d\bar{\Delta}_\perp \sim \hbar\omega\, d\bar{N}_\perp$):

$$\frac{d^3\bar{N}_\perp(t)}{\hbar\, d\omega\, dt\, d\Omega} = \frac{\alpha}{2\pi^3\hbar c}\mathrm{Im}\left\{\int_0^\infty \frac{\mu(\omega)\, dk}{\left[k^2 - \epsilon(\omega)\mu(\omega)\dfrac{\omega^2}{c^2}\right]}\right.$$

$$\left.\int_{-\infty}^{\infty} d\tau\ \left[k^2\mathbf{v}(t+\tau)\mathbf{v}(t) - \omega^2\right]\exp\left\{i\omega\tau - i\mathbf{k}[\mathbf{r}(t+\tau) - \mathbf{r}(t)]\right\}\right\}, \quad (2)$$

where α is the fine structure constant, $\hbar$ is the Planck's constant, $\mathbf{v}(t) = \dot{\mathbf{r}}(t)$ is the charge velocity, and k is the modulus of the photon wave vector $\mathbf{k}$. Ω is the solid angle defining the direction of $\mathbf{k}$ versus $\mathbf{v}(t)$.

- No radiation recoil effects on the charge trajectory, $\mathbf{r}(t)$.
- The condition, $\omega > 0$, results in $Im(...)$.
- Relation (2) can be integrated in respect of Ω for any $\mathbf{r}(t)$.

2 Cherenkov energy loss in infinite medium

For the case of movement with the constant velocity **v** the integral with respect to τ can be calculated ($\sim \delta(\mathbf{kv} - \omega)$):

$$\frac{d^3 \bar{N}_\perp(t)}{\hbar\, d\omega\, dt\, d\Omega} = \frac{\alpha}{2\pi^3 \hbar c} \mathrm{Im} \left\{ \int_0^\infty \frac{\mu(\omega) 2\pi k^2 v^2 \sin^2\theta\, \delta(\mathbf{kv} - \omega)\, dk}{\left[k^2 - \epsilon(\omega)\mu(\omega)\frac{\omega^2}{c^2}\right]} \right\},$$

and relation (2) is reduced to:

$$\frac{d^3 \bar{N}_\perp}{\hbar\, d\omega\, dx\, d\cos^2\theta} = \frac{\alpha}{\hbar c} \mathrm{Im} \left\{ \frac{\mu \tan^2\theta}{\pi\left[1 - \epsilon\mu\beta^2\cos^2\theta\right]} \right\}, \qquad (3)$$

where $\beta = v/c$, c is the speed of light in vacuum, $dx = v\, dt$ is the element of the particle trajectory and θ is the angle between **k** and **v**.

In the case of medium without magnetic permeability ($\mu = 1$, for optical frequencies) we get for the mean number of emitted Cherenkov photons from the unit particle trajectory length [3, 4, 5]:

$$\begin{aligned} \frac{d^3\bar{N}_\perp}{\hbar d\omega\ dx\ d\cos^2\theta} &= \frac{\alpha}{\hbar c}\frac{\Gamma\sin^2\theta}{\pi\left[\left(\cos^2\theta - \cos^2\theta_o\right)^2 + \Gamma^2\right]}, \\ \cos^2\theta_o &= \frac{\epsilon_1}{\beta^2|\epsilon|^2}, \quad \Gamma = \frac{\epsilon_2}{\beta^2|\epsilon|^2}. \end{aligned} \tag{4}$$

In the opposite case when $\epsilon = 1$ and the medium is defined by the complex magnetic permeability one gets:

$$\begin{aligned} \frac{d^3\bar{N}_\perp}{\hbar d\omega\ dx\ d\cos^2\theta} &= \frac{\alpha}{\hbar c}\frac{\Gamma_m\beta^{-2}\tan^2\theta}{\pi\left[\left(\cos^2\theta - \cos^2\theta_{om}\right)^2 + \Gamma_m^2\right]}, \\ \cos^2\theta_{om} &= \frac{\mu_1}{\beta^2|\mu|^2}, \quad \Gamma_m = \frac{\mu_2}{\beta^2|\mu|^2}. \end{aligned} \tag{5}$$

These relations clearly show that in media with absorption the angular distribution of Cherenkov radiation experience additional broadening. The distribution (4) has a sharp peak at the angle $\cos^2\theta_o = \epsilon_1/\beta^2|\epsilon|^2$, that can be considered as the most probable emission angle of Cherenkov photons in a non-transparent medium. The full width at half maximum of the angular peak, $FWHM \simeq 2\epsilon_2/\beta^2|\epsilon|^2$. Note that for small, but negative $\Gamma < 0$ ($\Gamma_m < 0$), the radiation has the tendency to be absorbed approaching the charge along directions with the angles around θ_o (θ_{om}) relative to the charge velocity $\mathbf{v}$.

In the optical-ultraviolet range the value of angular broadening can be estimated from: $\epsilon_2/\beta^2|\epsilon|^2 \sim c/n\omega l \sim \lambda/l \lesssim 10^{-5}$, where λ is the photon wave length, and l is the photon absorption length. It means that the broadening is in practice very difficult to observe in semi-transparent solids and liquids (Fig. 1).

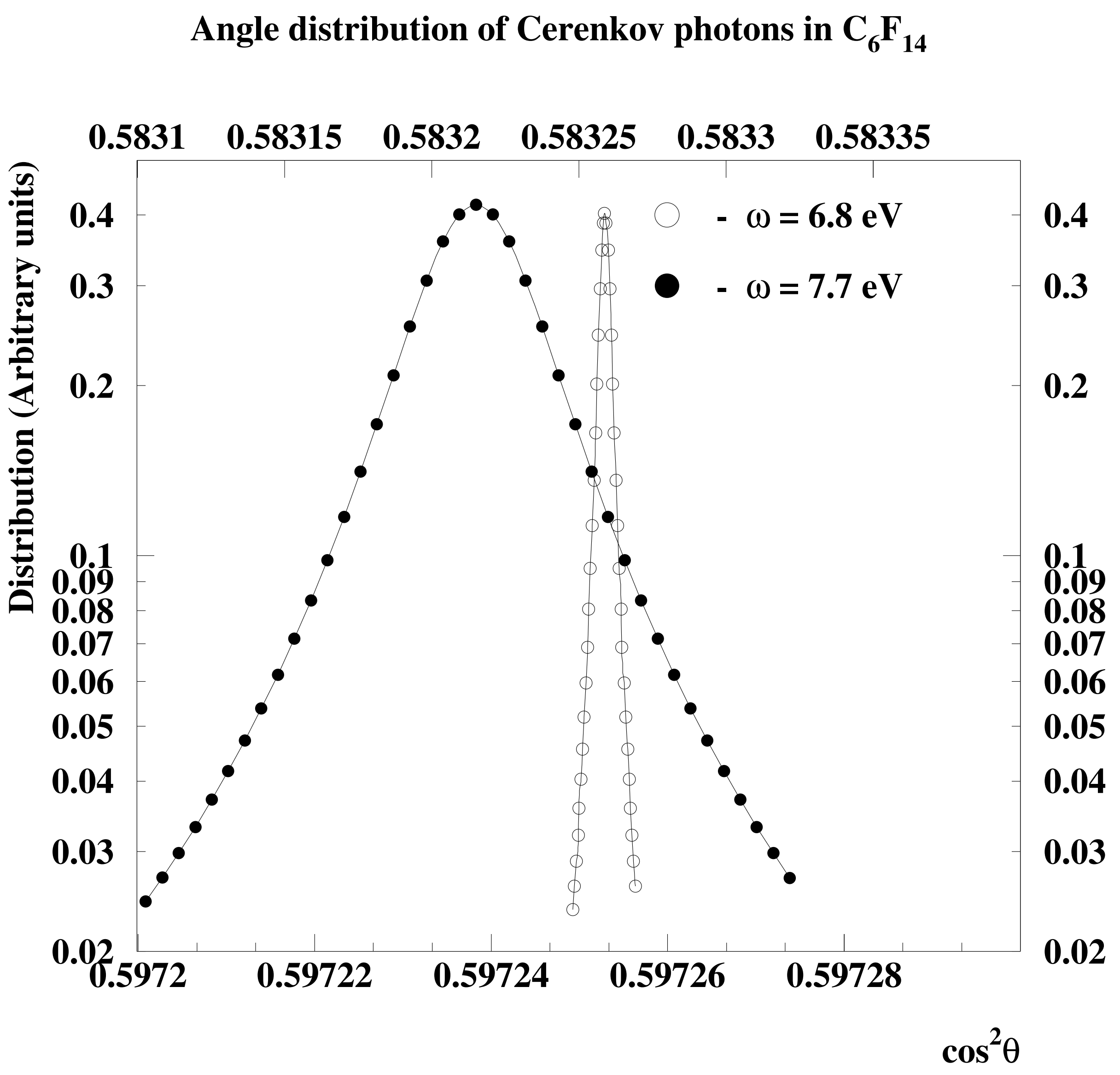

Fig. 1. The angle distributions of Cherenkov photons produced in C_6F_{14} by relativistic particle $v \sim c$ for two values of the photon energy: $\hbar\omega = 6.8$ eV (open circles) and $\hbar\omega = 7.7$ eV (closed circles and upper x-axis). The curves are normalised on the equal most probable values.

In the limit of a transparent medium, when $\epsilon_2 \to 0$ and $\mu_2 \to 0$, one can get the energy-angle distribution of the mean number of Cherenkov photons, $\bar{N}_c$ in the transparent medium with refractive index n ($n^2 = \epsilon_1\mu_1$):

$$\frac{d^3\bar{N}_c}{\hbar d\omega\ dx\ d\cos^2\theta} = \frac{\alpha\mu_1}{\hbar c}\left(1 - \frac{1}{\beta^2 n^2}\right)\cdot\delta\left(\cos^2\theta - \frac{1}{\beta^2 n^2}\right), \tag{6}$$

where δ is the Dirac delta function. This expression fixes explicitly the emission angle to be $\theta_c = \arccos(1/\beta n)$. If $\beta n > 1$, integration of (6) with respect to $\cos^2\theta$ results in the well known formula of the Frank-Tamm theory (in the particular case, $\mu_1 = 1$ and $n^2 = \epsilon_1$):

$$\frac{d^2\bar{N}_c}{\hbar d\omega dx} = \frac{\alpha}{\hbar c}\left(1 - \frac{1}{\beta^2 n^2}\right), \quad \beta n > 1. \tag{7}$$

One can consider another interesting case when $\epsilon_1\mu_1 > 0$, but both $\epsilon_1 < 0$ and $\mu_1 < 0$: Here the radiation follows the group velocity $\partial\omega/\partial\mathbf{k}$ which is anti-parallel to the wave vector $\mathbf{k}$. Therefore the radiation is emitted at the angle $\pi - \theta_c > \pi/2$ (see review [6]). However, relation (6) shows that Cherenkov radiation energy loss $\bar{\Delta}_c$, is negative ($\sim \mu_1$),

$$\frac{d^3\bar{\Delta}_c}{\hbar d\omega\ dx\ d\cos^2\theta} = \alpha\mu_1\frac{\omega}{c}\left(1 - \frac{1}{\beta^2\epsilon_1\mu_1}\right)\cdot\delta\left(\cos^2\theta - \frac{1}{\beta^2\epsilon_1\mu_1}\right), \tag{8}$$

$$\frac{d^2\bar{\Delta}_c}{\hbar d\omega dx} = \alpha\mu_1\frac{\omega}{c}\left(1 - \frac{1}{\beta^2\epsilon_1\mu_1}\right) < 0, \quad \beta^2\epsilon_1\mu_1 > 1.$$

and the radiation has the tendency to be absorbed along the direction $\pi - \theta_c$. Materials with both negative permittivity and permeability were recently experimentally investigated and it was observed the anomalous behaviour of refraction [7]. It would be interesting to investigate also the angular distribution of the Cherenkov radiation in this case taking in mind that the Cherenkov angle can be obtuse and to check that Cherenkov photons are absorbed by relativistic charge.

3 Doppler-like radiation

Let us consider the radiation of an oscillating dipole: the charge e moves according the law $\mathbf{r}_1(t) = \mathbf{v}t$, while the trajectory of the charge $-e$ is $\mathbf{r}_2(t) = \mathbf{v}t + \mathbf{a}\sin\omega_o t$. Here the amplitude $\mathbf{a}$ is supposed to satisfy the dipole approximation: $a = |\mathbf{a}| \ll v/\omega_0$. Then for $\mathbf{a} \parallel \mathbf{v}$ the radiation in a transparent medium (we put also, $\mu = 1$) reads ($d_\pm = \omega/(\omega \pm \omega_o)$):

$$\frac{d^3\bar{N}_\perp}{\hbar d\omega\, dx\, d\cos^2\theta} = \frac{\alpha}{\hbar c}\sin^2\theta \left\{ \left(\frac{a\omega}{2vd_+}\right)^2 \delta(\cos^2\theta - \cos^2\theta_+) + \right.$$

$$\left. + \left(\frac{a\omega}{2vd_-}\right)^2 \delta(\cos^2\theta - \cos^2\theta_-) \right\}, \qquad (9)$$

where the first and second terms are responsible for the anomalous and normal Doppler dipole radiations in a transparent medium, respectively:

$$\cos\theta_\pm = \frac{\omega \pm \omega_o}{\beta n \omega} = \left(1 \pm \frac{\omega_o}{\omega}\right)\cos\theta_c.$$

The Cherenkov angle $\cos\theta_c = 1/\beta n$ separates the regions into anomalous:

$$\omega = \frac{\omega_o}{\beta n \cos\theta - 1}, \quad \text{for,} \ \cos\theta > \cos\theta_c,$$

and normal:

$$\omega = \frac{\omega_o}{1 - \beta n \cos\theta}, \quad \text{for,} \ \cos\theta < \cos\theta_c,$$

Doppler effects [5]. The oscillating dipole like radiation can be experimentally observed by passing powerful laser pulses through a medium [8]. Here ω_o approximately corresponds to the carrying frequency of the laser pulse. Since the threshold for normal Doppler effect can be essentially smaller than for the Cherenkov radiation, the radiation can be observed at subluminal velocities $\beta < 1/n$. Here $\cos\theta_c > 1$, while the observable line of the normal Doppler effect emits at $\cos\theta_- < 1$.

4 Relativistic rise of ionisation

In the range of atomic frequencies Cherenkov energy loss does not create practically of observable photons. Here the absorption is high and all emitted photons will be absorbed in the vicinity of the particle trajectory resulting in observable (in special detector media) ionisation. PAI-model [9, 10] reads:

$$\frac{d^2\bar{N}_\perp}{\hbar d\omega\ dx} = \frac{\alpha}{\pi\hbar c}\mathrm{Im}\left\{\left[1-\frac{1}{\beta^2\epsilon}\right]\ln\left(\frac{1}{1-\beta^2\epsilon}\right)\right\}. \tag{10}$$

and

$$\frac{d^2\bar{N}_\parallel}{\hbar d\omega\, dx} = \frac{\alpha N}{\pi\hbar\beta^2\omega}\left[\sigma_\gamma(\omega)\ln\frac{2mv^2}{\hbar\omega} + \frac{1}{\omega}\int_0^\omega \sigma_\gamma(\omega')\, d\omega'\right], \tag{11}$$

where m is the electron mass, N is the atomic density, and $\sigma_\gamma(\omega)$ is the photo-absorption cross-section. In many applications for particle identification it is interesting to investigate relativistic dependences of the ionisation energy loss distribution parameters, f.e. the most probable energy loss (Fig. 2).

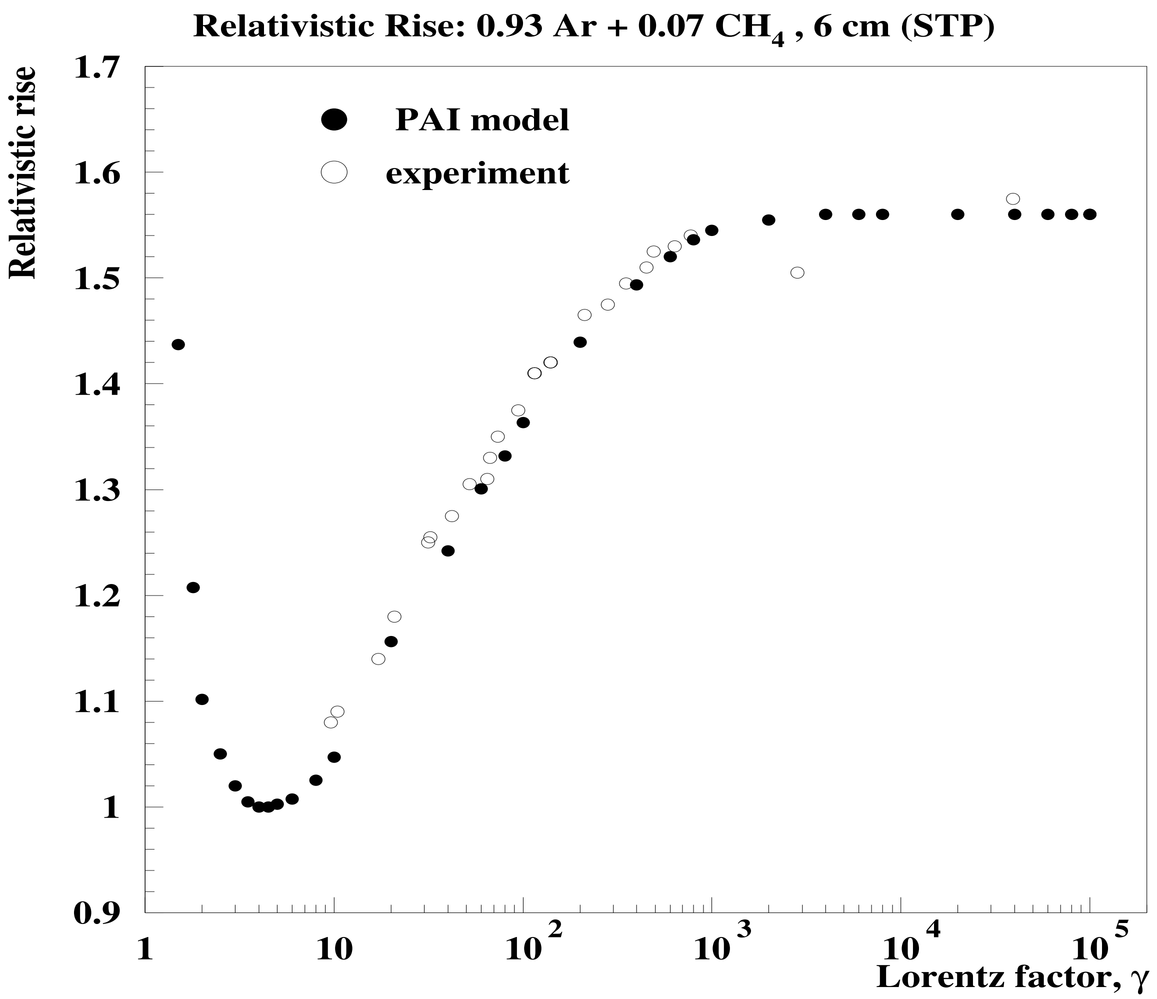

Fig. 2. Relativistic rise of the most probable ionisation energy loss in the gas mixture$93\% Ar + 7\% CH_4$ with the thickness of 6 cm (STP). Open circles are the experiment [11], closed circles are GEANT4 simulation according to the PAI model.

5 Radiation in the X-ray range

For the energy transfers more than K-shell excitation potential we can use the standard high frequency approximation for the dielectric permittivity:

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2} + i\frac{c}{\omega l}, \tag{12}$$

where ω_p and l are the plasma frequency and the photon absorption length in the medium, respectively. Then the mean number of X-ray Cherenkov radiation (XCR) photons emitted from the unit distance will be [12] (the small angle approximation of (4)):

$$\frac{d^3\bar{N}_{xcr}}{\hbar d\omega\, dx\, d\theta^2} = \frac{\alpha}{\pi\hbar c}\frac{\omega}{c}\theta^2 \mathrm{Im}\left\{Z\right\}, \tag{13}$$

where we introduce the *complex formation zone*, Z, of XCR in the medium:

$$Z = \frac{L}{1 - i\dfrac{L}{l}}, \quad L = \frac{c}{\omega}\left[\gamma^{-2} + \frac{\omega_p^2}{\omega^2} + \theta^2\right]^{-1}, \quad \gamma^{-2} = 1 - \beta^2. \tag{14}$$

In the case of a transparent medium $l = \infty$, the complex formation zone is reduced to the *coherence length* L of XCR.

Since usually $\omega_p^2/\omega^2 \gg c/\omega l$, the number of emitted XCR photons is considerably suppressed (and disappears in the limit of transparent medium) by the destructive interference between the photons emitted from different parts of the particle trajectory.

For the case of undulator-like radiation $\mathbf{r}(t) = \mathbf{v}t + \mathbf{a}\sin\omega_o t$ the X-ray radiation will be defined by the normal Doppler effect. Neglecting medium absorption in the X-ray range, one has:

$$\frac{d^2\bar{N}}{\hbar d\omega\, dx} = \frac{\alpha}{\hbar c}\frac{a^2\omega^2}{4v^2}\left(1-\frac{\omega_o}{\omega}\right)^2\left[2\frac{\omega_o}{\omega}-\gamma^{-2}-\frac{\omega_p^2}{\omega^2}\right]. \tag{15}$$

X-ray radiation can be observed for frequencies satisfying:

$$\gamma\omega_p \lesssim \omega \lesssim \gamma^2\omega_o, \quad \omega_p \ll \gamma\omega_o.$$

For the case of prompt bremsstrahlung $0 \to \mathbf{v}$, the number of X-ray photons $\bar{N}_{xbr}$ produced by ultra-relativistic $\beta \sim 1$ charge reads:

$$\frac{d^2\bar{N}_{xbr}}{\hbar d\omega\, d\theta^2} = \frac{\alpha}{\pi\hbar c}\frac{\omega}{c}\theta^2 \mathrm{Re}\left\{Z^2\right\}. \tag{16}$$

Note that this relation is quite similar to the the mean number of X-ray transition radiation photons $\bar{N}_{xtr}$, emitted when the charge e crosses the interface between two media with different dielectric properties:

$$\frac{d^2\bar{N}_{xtr}}{\hbar d\omega\, d\theta^2} = \frac{\alpha}{\pi\hbar c}\frac{\omega}{c}\theta^2 \mathrm{Re}\left\{(Z_1-Z_2)^2\right\}. \tag{17}$$

6 X-ray transition radiation

The destructive interference of X-ray Cherenkov radiation can be destroyed, if the charge crosses an interface between two media with different dielectric permittivities. The additional work produced by the charge against electric fields induced near the interface results in additional radiation which is called as X-ray transition radiation (XTR).

Using the methods developed in ref. [13] one can derive the relation describing the mean number of XTR photons generated per unit XTR photon frequency and θ^2, $d^2\bar{N}_{in}/\hbar d\omega\, d\theta^2$, *inside* the radiator for the most general XTR radiator consisting of n absorbing media with fluctuating thicknesses (fig. 3):

$$\begin{aligned} \frac{d^2\bar{N}_{in}}{\hbar d\omega\, d\theta^2} \quad &= \quad \frac{\alpha}{\pi\hbar c^2}\omega\theta^2 \mathrm{Re}\left\{\sum_{i=1}^{n-1}(Z_i - Z_{i+1})^2 + \right. \\ &+ \quad \left. 2\sum_{k=1}^{n-2}\sum_{i=1}^{n-k-1}(Z_i - Z_{i+1})(Z_{i+k} - Z_{i+k+1})\prod_{j=1}^{k} F_{i+j}\right\}. \qquad (18) \end{aligned}$$

General XTR Radiator

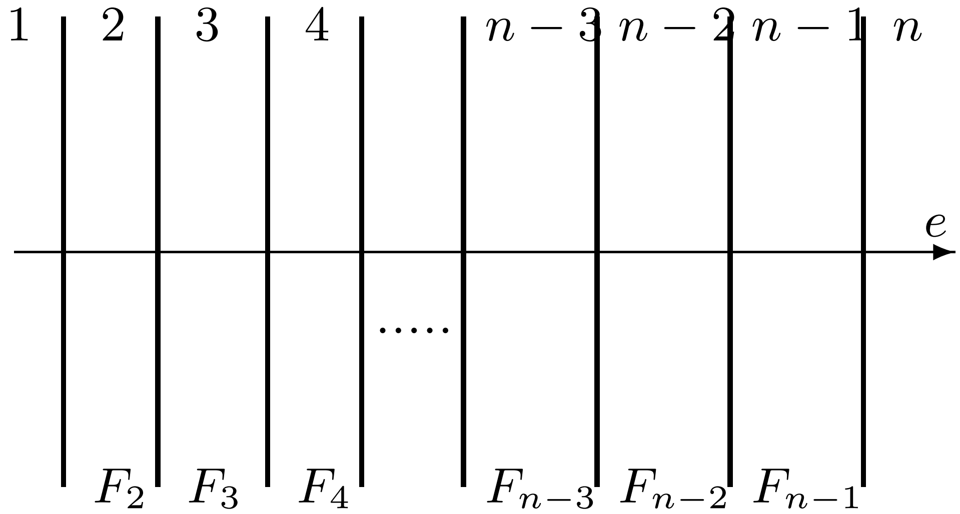

Fig. 3. Diagram of a charged particle crossing the most general XTR radiator consisting of n media and $n-1$ interfaces between them. The medium thicknesses t_k can fluctuate separately as described by the F_k $(k = 2, 3, \ldots, n-1)$ values.

In the case of gamma distributed gap thicknesses the values F_j, $(j = 1, 2)$ are:

$$F_j = \int_0^\infty dt_j \left(\frac{\nu_j}{\bar{t}_j}\right)^{\nu_j} \frac{t_j^{\nu_j - 1}}{\Gamma(\nu_j)} \exp\left[-\frac{\nu_j t_j}{\bar{t}_j} - i\frac{t_j}{2Z_j}\right] = \left[1 + i\frac{\bar{t}_j}{2Z_j\nu_j}\right]^{-\nu_j}, \quad (19)$$

where Z_j is the complex formation zone of XTR (similar to XCR, relation (14)) in the j-th medium [13, 14], Γ is the Euler gamma function, $\bar{t}_j$ is the mean thickness of the j-th medium in the radiator and $\nu_j > 0$ is the parameter roughly describing the relative fluctuations of t_j. In fact, the relative fluctuation is, $\delta_j = 1/\sqrt{\nu_j}$.

In the particular case of n foils of the first medium interspersed with gas gaps of the second medium, one obtains:

$$\frac{d^2\bar{N}_{in}}{\hbar d\omega\, d\theta^2} = \frac{2\alpha}{\pi\hbar c^2}\omega\theta^2 \mathrm{Re}\left\{\langle R^{(n)}\rangle\right\}, \quad F = F_1 F_2, \quad (20)$$

$$\langle R^{(n)}\rangle = (Z_1 - Z_2)^2 \left\{ n\frac{(1-F_1)(1-F_2)}{1-F} + \frac{(1-F_1)^2 F_2[1-F^n]}{(1-F)^2} \right\}. \quad (21)$$

This approach allows to implement XTR as GEANT4 parametrisation [14], or as standard electro-magnetic process.

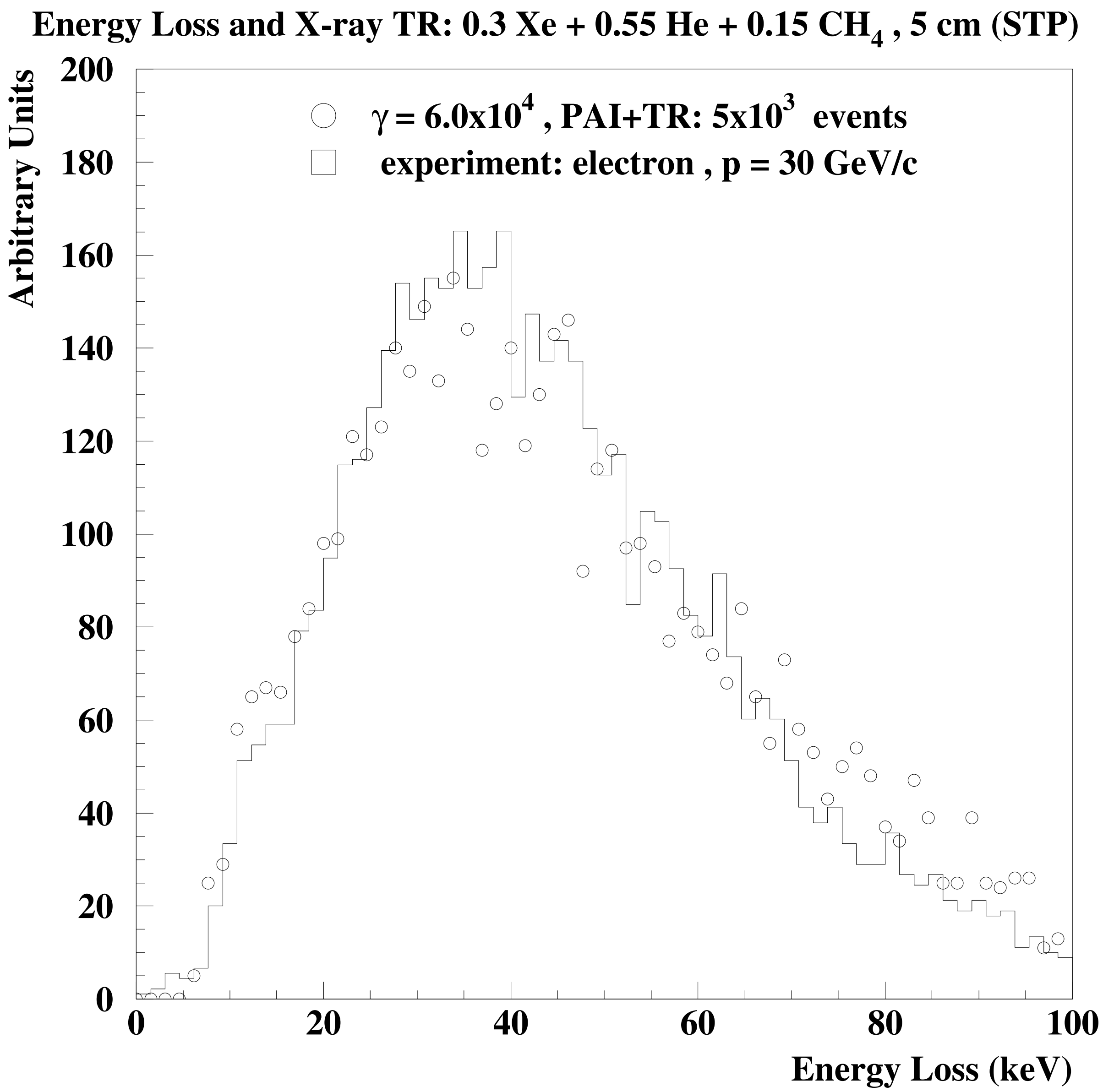

Fig. 4. The total energy loss distribution produced by electrons with a momentum of 30 GeV/c in a gas mixture $0.35\mathrm{Xe} + 0.55\mathrm{He} + 0.15\mathrm{CH}_4$, 5 cm thick at 1 atm. The histogram is the experiment [15], open circles are GEANT4 simulation. XTR radiator: $350 \cdot (\mathrm{CH}_2/\mathrm{CO}_2 = 0.019/0.6)$ mm.

7 Conclusions

In the framework of GEANT4 [1] radiation processes in matter were implemented in the following classes:

1. G4CerenkovRadiation for the description of Cherenkov radiation. The aberration due absorption was not still implemented because of lack of reliable optical data for the absorption in ultraviolet range.

2. G4SandiaTable, G4StaticSandiaData, G4PAIxSection, and G4PAI(Photon)Model for the description of the PAI ionisation model (with secondary photons).

3. The X-ray transition radiation as electro-magnetic process is described by : G4VXTRenergyLoss, G4RegularXTRadiator and tunable G4GammaXTRadiator.

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