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Radiation of superluminal sources in vacuum

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Abstract

Vavilov–Cherenkov radiation is emitted by a charged body uniformly moving in a medium when the body velocity exceeds that of light in the medium. Therefore, it was believed that Vavilov–Cherenkov radiation is impossible in vacuum, because the velocity of any material body cannot exceed the light velocity in vacuum. However, it is possible to realize distributions of charges and currents which propagate with any given velocity. Such a superluminal distribution can be used as a source of Vavilov–Cherenkov radiation in vacuum.

Keywords: Vavilov–Cherenkov radiation; Superluminal velocity; Superluminal source of radiation

A source uniformly moving in a medium radiates directed waves if the source velocity exceeds the velocity of waves in that medium. A particular manifestation of this general law was recognized a long time ago. Apparently this phenomenon was firstly investigated in hydrodynamics with an example of surface waves excited by a moving ship. Later on, Ernest Mach investigated sound waves generated by a projectile moving in air with the supersonic velocity. Mach showed that these waves propagate at an angle θ with respect to the projectile velocity. Here $\cos\theta = u/v$, where u is the sound velocity and v is the projectile velocity. Mach succeeded to take photographs of conic waves radiated in this case. Mach's experiments got high recognition of physicists that time. This can be seen in particular from an article of Einstein (1916) devoted to Mach.

In an analogous way it could be supposed that a similar phenomenon takes place in electrodynamics. Indeed, a charged object moving in a medium faster than electromagnetic waves in that medium becomes a source of directed electromagnetic radiation. Understanding of this fact came, however, much later. Apparently, there are some reasons for that. One of them is that the speed of light in vacuum, as well as that in a refracting media, is so large that it would be difficult to imagine a material object moving faster than light. So, an article of Heaviside (1887) where this possibility had been investigated did not attract attention of physicists. The second reason follows from the theory of relativity, according to which the velocity of a material body cannot exceed the speed of light. So, the possibility of superluminal velocity was under doubt. The relativistic theory forbids motion of material bodies with the velocity that is larger than the light velocity in vacuum, which is about 300 000 km/s. In a refractive medium, the light velocity is usually much less. For example, the light velocity is about 200 000 km/s in a glass with the refractive index of 1.5. That is the velocity of an electron having the energy of about 700 keV. Electrons of higher energies move faster than light in the glass. This was fully realized after the article of Tamm and Frank (1937) explaining results of experiments described by Vavilov (1934) and Cherenkov (1934).

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The theory of Vavilov–Cherenkov radiation is elaborated now fully enough—perhaps in more details than theories of analogous phenomena in other areas of physics are (for example, the theory of the Mach effect). This is explained by practical importance of this phenomenon for high-energy physics as well as the fact that investigations of similar phenomena in hydrodynamics and acoustics are much more difficult because of a strong influence of non-linear processes.

Sommerfeld (1904) investigated the electromagnetic field of a particle moving in vacuum faster than light. He showed that radiation appears in this case with a sharp angular distribution. Emitted waves are directed at an angle θ with respect to the velocity of the charged particle, where $\cos\theta = u/v$, u is the light velocity in vacuum, and v is the particle velocity. Soon after 1904, special relativity was born. Superluminal velocity was understood to be forbidden for material bodies and Sommerfeld's article was forgotten for a long time. Nevertheless, much later, the understanding came that it is possible to produce a radiation source moving faster than the light velocity in vacuum. Apparently, the first model of such a source was the model of Frank (1942). We present essentials of his article below.

Consider a light pulse that consists of plane electromagnetic waves. Suppose that the field is not zero between two parallel planes and is zero in the rest of the space. The pulse propagates in the medium having the dielectric constant ε_1 to the plane boundary of another medium having the dielectric constant ε_2. The geometry of the problem is shown in Fig. 1. The front of the incident wave is marked with 1 in that figure. The angle of the light pulse is θ_0. The pulse velocity in the upper medium is denoted as $v_1 = c/\sqrt{\varepsilon_1}$. It is easy to notice that the area where the impulse intersects the boundary surface ("the spot") moves along the boundary with the velocity

$$v = \frac{v_1}{\sin\theta_0} = \frac{c}{\sqrt{\varepsilon_1}\sin\theta_0}. \tag{1}$$

If $\varepsilon_1 = 1$, the velocity of the "spot" is bigger than the light velocity in vacuum. This is not in contradiction with special relativity, because at every moment the spot is created by a new portion of the pulse. Nevertheless, the pulse induces a real current and charge density within the spot area. These induced charges and currents move along the boundary together with the spot. So, generally speaking, we get a radiation source, the velocity of which exceeds the light velocity in the first medium. Such a source has to emit Vavilov–Cherenkov radiation. The Vavilov–Cherenkov wave is marked with 2 in the figure. The angle $\theta_{1\mathrm{V-Ch}}$ between the direction of wave 2 and the spot velocity is described by the formula

$$\cos\theta_{1\mathrm{V-Ch}} = \frac{c}{\sqrt{\varepsilon_1}v}. \tag{2}$$

Placing the value v from Eq. (1) to formula (2), we get

$$\cos\theta_{1\mathrm{V-Ch}} = \frac{c}{\sqrt{\varepsilon_1}}\frac{\sqrt{\varepsilon_1}\sin\theta_0}{c} = \sin\theta_0. \tag{3}$$

It is not difficult to deduce that wave 2 propagates away from the boundary and its direction of propagation makes the angle $\theta_1 = \theta_0$ with the normal to the boundary. So, wave 2 is actually a reflected wave while pulse 1 belongs to the incident wave.

Let us consider the field on the other side of the boundary, i.e. in the medium with the dielectric constant ε_2. The spot moving along the boundary with velocity (1) can become a radiation source in the second medium only in the case when the spot velocity is larger than the light velocity in the second medium, i.e. if the following inequality is satisfied:

$$v = \frac{c}{\sqrt{\varepsilon_1}\sin\theta_0} > \frac{c}{\sqrt{\varepsilon_2}}. \tag{4}$$

In this case, the Vavilov–Cherenkov wave appears in the second medium. It is marked with 3 in Fig. 1. The angle between its direction of propagation and the spot velocity is determined by the formula

$$\cos\theta_{2\mathrm{V-Ch}} = \frac{c}{\sqrt{\varepsilon_2}v} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\sin\theta_0. \tag{5}$$

If we introduce the angle $\theta_2 = (\pi/2) - \theta_{2(\mathrm{V-Ch})}$ (the refraction angle), then we get the well known relation

$$\sqrt{\varepsilon_1}\sin\theta_0 = \sqrt{\varepsilon_2}\sin\theta_2. \tag{6}$$

This is nothing but Snell's law. Thus, Vavilov–Cherenkov radiation generated by the spot in the second medium coincides with the refraction wave. If the spot velocity is less than the light velocity in the second

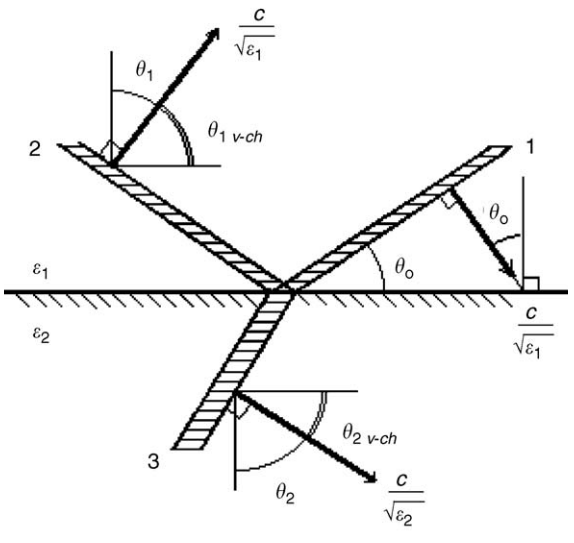

Fig. 1. Reflection and refraction considered as the Vavilov–Cherenkov effect. Geometry of the problem: 1: incident wave, 2: reflected wave, 3: refracted wave.

medium, i.e. if

$$v = \frac{c}{\sqrt{\varepsilon_1}\sin\theta_0} < \frac{c}{\sqrt{\varepsilon_2}}, \tag{7}$$

then the Vavilov–Cherenkov wave in the second medium is not excited and the incident pulse 1 does not penetrate into the second medium. It is obviously that inequality (7) is equivalent to the condition of total reflection

$$\sin\theta_0 > \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \equiv \sin\theta_{\mathrm{refl}}. \tag{8}$$

Thus, reflection and refraction of waves on the boundary can be regarded as Vavilov–Cherenkov radiation of charge and current distributions induced by the incident wave on the boundary surface.

Let us consider now the case when the light ray reflected from a mirror falls on the boundary between two media and creates the aforesaid "spot" on the boundary. When the mirror rotates, this spot moves along the boundary surface and, under some conditions, can act as a source of directed radiation.

Consider the field created by a rotating light source (Bolotovskii and Zhemanova, 1985). Let us introduce the cylindrical coordinate system r, φ, z, where z is the axis of the cylindrical system, r is the distance from the chosen point to the axis, and φ is the azimuth. Suppose that z is the axis of a hollow cylinder of radius a. The cylinder surface is transparent in the angular interval $\varphi < |\alpha|$ and opaque outside the interval. Suppose that the light source is located at the z-axis. Fig. 2 gives the geometry of the problem in the plane perpendicular to the z-axis. Let the cylinder rotate around the z-axis with the angular velocity Ω. Obviously, the model reminds of a rotating beacon. And transparent part of the cylinder plays here a role of the source aperture. Let ω be the frequency of light emitted by the source located at the z-axis. Then the electric field E on the cylinder surface ($r = a$) will be described by the following expression:

$$E|_{r=a} = \begin{cases} E_0\exp[-\mathrm{i}\omega t] & \text{when} |\varphi - \Omega t| < \alpha, \\ 0 & \text{on the rest surface.} \end{cases} \tag{9}$$

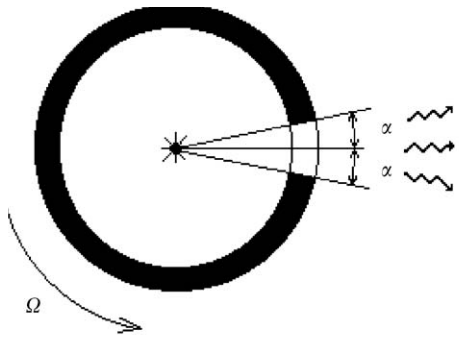

Fig. 2. Rotating beacon.

The field E on the circle $r = a$ may be represented by the Fourier series

$$E_{r=a} = \sum_n E_n \exp[-\mathrm{i}\omega t - \mathrm{i}n(\varphi + \Omega t)]. \tag{10}$$

Taking into account the cylindrical symmetry of the problem, the field at large distances from the z-axis will be

$$E = \sum_n E_n \exp[-\mathrm{i}n\varphi - \mathrm{i}(\omega + n\Omega)t] \times \frac{1}{\sqrt{r}}\exp\left[\mathrm{i}\left(\frac{\omega + n\Omega}{c}\right)r\right]. \tag{11}$$

It can be seen from this formula that the field consists of diverging waves of frequencies determined by the initial wave frequency ω as well as by the cylinder rotation frequency Ω. Surface of constant phase for these waves will be

$$-n\varphi - \omega_n t + \frac{\omega_n}{c}r = \mathrm{const} \tag{12}$$

with $\omega_n = \omega + n\Omega$. At a given moment of time, Eq. (12) describes an Archimedes' spiral. The distance between two consecutive turns of the spiral is $n\lambda_n$, where $\lambda_n = 2\pi c/\omega_n$ and λ_n is the wavelength corresponding to the nth harmonic.

Consider now a remote cylindrical surface at a large distance R from the source (see Fig. 3). Choose part of the surface corresponding to a small interval of azimuth angles $\mathrm{d}\varphi$. It is seen from the figure that the angle ψ between the circle $r = R$ and the surface of constant phase is determined by the formula

$$\psi \simeq \tan\psi = \frac{\mathrm{d}r}{r\,\mathrm{d}\varphi} = \frac{cn}{r\omega_n}. \tag{13}$$

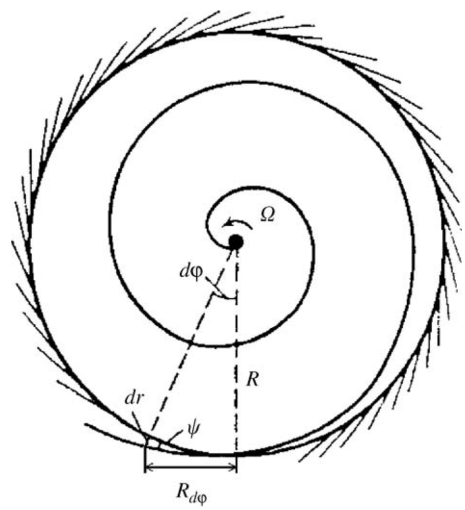

Fig. 3. Cylindrical surface of the wave front at the distance R from the source.

Here the ratio $dr/d\varphi$ has been determined from Eq. (12) for the surface of constant phase. We can consider the expanding wave inside the surface $r = R$ as an incident wave. Then ψ in Eq. (13) is the angle of incidence (the hade). Since the curvature of the wave can be ignored at large distances from the source and for small enough parts of the surface $r = R$, a physical picture of the phenomenon is the same as in the case of a plane wave incident on the plane boundary (see Fig. 1).

It follows from formula (13) that ψ at large distances from the source can be arbitrarily small. At the same time the angle ψ is nothing but the angle of incidence of the wave on the surface. So, the incident wave excites a charge and current density in the region where the surface of constant phase intersects the screen surface. This region ("spot") moves along the screen surface $r = R$, and we may use Eq. (1) to determine its velocity:

$$v \simeq \frac{c}{\psi} = \frac{r\omega_n}{n}. \tag{14}$$

It follows from this formula that the velocity of the "spot" created by the rotating ray is proportional to the distance between the screen and the light source (z-axis). The spot velocity exceeds the speed of light for distances large enough. Then the rotating "spot" becomes a radiation source. Note that this radiation source is excited in every moment of time by different parts of the wave front as it was in the case of a plane wave incident on the plane boundary.

Let us consider now another example of a superluminal radiator, where general characteristic features of the phenomenon can be clearly seen. The general feature resides in the fact that the radiating area moves with a superluminal velocity; however, each impulse of radiation is generated by a new particle (Bolotovskii and Ginzburg, 1972; Bolotovskii, 1972).

Consider an ideally conducting plane using a rectangular coordinate system x, y, z, in which the location of the conducting plane is described by $z = 0$ (Fig. 4). Thus, the z-axis is perpendicular to the plane. Suppose that an electron moving along the z-axis intersects the surface at the point $x = 0$ at the time moment $t = 0$. Herewith, a flash of transition radiation (TR) is generated. The field of TR at large distances from the point $x = 0$ can be represented in the form

$$\mathbf{E}_1 = \mathbf{E}(\mathbf{k})e^{i\mathbf{kr}-i\omega t}. \tag{15}$$

Suppose further that, after a time interval T, another electron having the same velocity v hits the surface at the point $x = L$. Another flash of TR is generated, and its field at large distances from the point $x = L$ is

$$\mathbf{E}_2 = \mathbf{E}(\mathbf{k})e^{i\mathbf{k}(\mathbf{r}-\mathbf{L})-i\omega(t-T)}. \tag{16}$$

Suppose that this process is repeated periodically, i.e. the nth electron hits the surface at the point $x = nL$ at the time moment $t = nT$, and so on. The total radiation field is equal to the sum of the fields $\mathbf{E}_1, \mathbf{E}_2, \ldots, \mathbf{E}_n$. If the number of the particles is infinite, we get after summation

$$\mathbf{E} = \sum_{0}^{N\to\infty} \mathbf{E_n}(\mathbf{k}) = 2\pi\mathbf{E}(\mathbf{k})e^{i\mathbf{kr}-i\omega t} \times\delta(\mathbf{kL} - \omega T - 2\pi m), \tag{17}$$

where m is an arbitrary integer and $\delta(x)$ is the Dirac delta-function. Hence, the total field of TR differs by the factor $\delta(\mathbf{kL} - \omega T - 2\pi m)$ from the field of TR emitted by a single charge. Therefore, an additional condition arises which is imposed on the radiation field. The condition consists in equating the argument of the delta function to zero. It means that only the waves for which the condition is satisfied remain in TR. Taking into account the geometry of the problem, we may rewrite this condition in the form

$$\omega = \frac{2\pi m}{T(1 - (L/cT)\cos\theta)}, \tag{18}$$

where θ is the angle between the x-axis and the direction of radiation.

Emitted radiation can be considered as radiation generated by the blinking source which moves along the x-axis with the velocity $v = L/T$ and produces flashes with the period T. If $m = 0$ in Eq. (18), then radiation with a frequency ω can exist only if the following condition is fulfilled:

$$1 - \frac{L}{cT}\cos\theta = 0. \tag{19}$$

So, for $m = 0$ radiation exists only on the surface of the cone with the apex angle θ. The axis of the cone coincides with the x-axis. Hereby, the angle of radiation θ is determined by the relation $\cos\theta = c/v$. This relation is, formally speaking, the condition for generation of Vavilov–Cherenkov radiation in vacuum. The value $v = L/T$ may exceed the light velocity without a violation of special relativity. For $m \neq 0$, condition (18) determines a

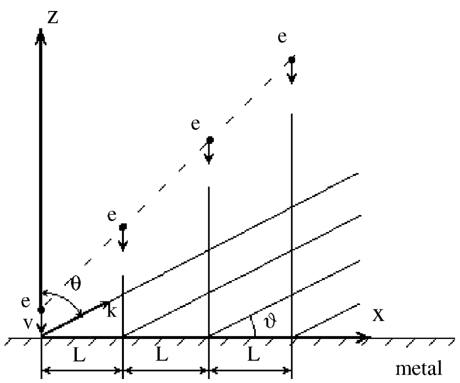

Fig. 4. The model of the blinking moving source.

set of frequencies typical for the Doppler effect. Thus, in the problem under consideration, the TR field contains factors characteristic for Vavilov–Cherenkov radiation and for the Doppler effect.

If sum (17) contains a finite number N of terms (or, in other words, the number of particles which produce TR flashes in series is finite), the result of summation becomes proportional to

$$S_N = \frac{1 - e^{iN\Delta}}{1 - e^{i\Delta}} = \frac{\sin N\Delta/2}{\sin \Delta/2} e^{i((N-1)\Delta/2)}, \quad (20)$$

where N is the number of colliding particles and $\Delta = \omega T - \mathbf{kL}$. In this case, radiation has a series of maxima instead of one maximum for a single charge.

Using the above mentioned system of colliding particles, it is possible to create a source of Vavilov–Cherenkov radiation in a waveguide. The phase velocity of waves in a waveguide is larger than the light velocity in vacuum. Hence, real charged particles that move in a waveguide cannot emit Vavilov–Cherenkov radiation themselves. However, a moving source of Vavilov–Cherenkov radiation in a waveguide still can be constructed (Afanassyev and Bolotovskii, 1972; Afanassyev, 1975). Let us consider, for simplicity, a rectangular waveguide. Suppose that a charged particle, velocity of which is perpendicular to the axis of the waveguide, intersects it at some cross section. After the period T, another charged particle with the same velocity intersects the waveguide at another cross section at the distance L from the first cross section. Then, at the moment $2T$, the third charged particle intersects the waveguide at the distance $2L$ from the first cross section, and so on. Each intersection is accompanied with a flash of TR in the waveguide. The whole picture looks as if some source of radiation moves in the waveguide along the axis with the velocity $v = L/T$. If v coincides with the phase velocity of a certain proper wave in the waveguide, generation of the corresponding harmonic appears. In this case, as well as in the above-described problem, it is possible to achieve Vavilov–Cherenkov radiation as well as the Doppler effect.

A source of radiation in a waveguide can also be made with the help of a continuous beam of charged particles if the beam intersects the waveguide in such a way that the point of intersection moves along the waveguide. The portion of beam which is inside the waveguide moves as a whole along the axis. Its velocity may well exceed the light velocity in vacuum. If the velocity equals to a phase velocity of a certain proper wave in the waveguide, then generation of this wave takes place. This possibility was used in a device known as gyrocon (Budker et al., 1979), a powerful generator of radio waves in centimeter bands. A scheme of the gyrocon is shown in Fig. 5. An accelerating device 1 prepares a beam of relativistic electrons. The beam passes through

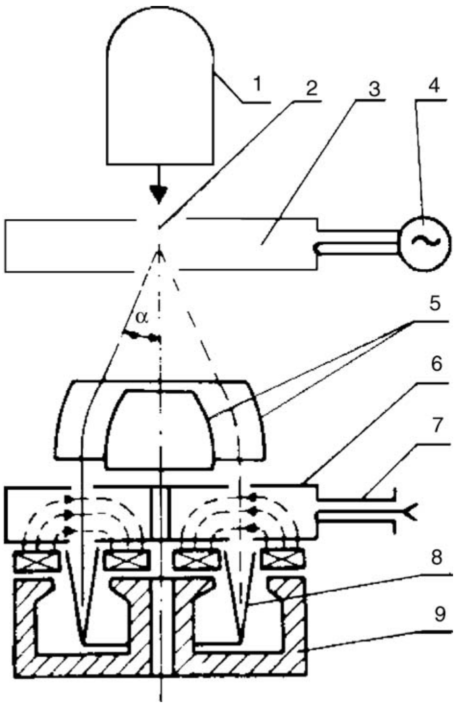

Fig. 5. The scheme of a gyrocon: (1) HV accelerator, (2) electron beam, (3) scanning resonator, (4) HF exciter, (5) electrostatic deflection system, (6) output resonator, (7) energy outputs, (8) collector, (9) compensating electromagnet. α is the deflecting angle.

a resonator 4 with a rotating electromagnetic field. After leaving the resonator, the beam trajectory looks like an untwisting spiral laid on a cone surface with a deviation angle α. Then the beam enters an electrostatic deflection system 5. After passing the deflection system, the bunch of particles moves parallel to the initial velocity, but the beam trajectory is shifted from the axis and the beam is rotating around the axis as a whole. Then the beam enters the output resonator 6 which is a waveguide coiled up (like a torus ring). Part of the beam located inside the resonator moves along a circular orbit. The frequency of revolution is selected in such a way that a portion of the beam which is inside the coiled up waveguide moves along the waveguide with the velocity equal to the phase velocity of the wave to be generated. If the linear velocity of such a source coincides with the phase velocity of one of the proper harmonics of the output resonator then generation of this harmonic takes place. Phase velocities of proper harmonics in a waveguide are larger than the light velocity. In particular, Budker et al. (1979) mentioned that they used gyrocon for generation of a harmonic having the phase velocity 1.84 times the speed of light. Gyrocon operating in a DC regime produces radiation in the wavelength range from 30 cm to 1.1 m. The power of generated radiation achieves 5 MW, with the efficiency of bunch energy transformation into radiation achieving 80%.

A radiation source moving along an arbitrary trajectory with any velocity can be created with the help of a collimated beam of charged particles. Consider a well-collimated beam of charged particles incident upon a conducting plane. The point where the beam intersects the plane is the source of TR. This point (or spot) can be forced to move along the plane with the help of deflection system, and the motion velocity can be made larger than the light velocity. For example, it is possible to satisfy conditions under which the spot will move uniformly along a circular orbit (Maneva, 1975, 1977). In this case generated radiation has much in common with synchrotron radiation. In particular, radiation spectrum consists of frequencies that are multiples of the revolution frequency. However, there are also important differences. Velocity of the spot on the plane can exceed the light velocity while this is impossible for material particles. In similar way the superluminal undulator can also be considered (Afanassyev, 1974). Note that a moving spot formed by a light ray incident upon the conducting plane may also be regarded as a radiation source, as well as the spot produced by a beam of charged particles.

In this regard it is necessary to mention that properties of radiation depend not only on the character of the spot motion but also on changes in the size and form of the spot. In order to get some idea of peculiarities which are characteristic for TR generated by an extended bunch, we have considered TR from a spherical bunch with a uniform charge density (Bolotovskii and Serov, 2002). Suppose that the spherical bunch moves uniformly with a velocity v along the z-axis. Radius of the bunch is denoted by r_0. Let the boundary coincide with the xy-plane. Suppose that the bunch touches the boundary at $t = 0$. Let us divide the volume of the bunch into thin layers parallel to the boundary. During the passage of the bunch these layers intersect the boundary in series and generate TR. The interference of the waves emitted by every layer gives TR from the whole bunch. Every layer is a circle with a uniform charge density. Radius $a(t)$ of the circular layer situated on the boundary at the instant t changes during the passage of the bunch. Initially, radius of the circle is zero and it becomes

$$a(t) = \sqrt{r_0^2 - (r_0 - vt)^2} \tag{21}$$

at the moment t. Evolution of the radiating area is determined by changes of the circle radius with time. During the passage of the spherical bunch through the plane $z = 0$ the radiating circle radius changes from zero up to radius of the bunch and then decreases to zero. The velocity that describes the expansion (and subsequent diminution) of the radiating area is equal to

$$\frac{\mathrm{d}a}{\mathrm{d}t} = v\frac{r_0 - vt}{\sqrt{2r_0vt - v^2t^2}}. \tag{22}$$

Thus, the area that generates TR arises and expands (with a superluminal velocity during some time) and then collapses and disappears. If the velocity v_0 of the bunch is close to the light velocity, then, in the time interval from $t = 0$ up to $t \simeq 0.3r_0/c$, the velocity of the expansion exceeds that of light and, in the time interval from $t \simeq 1.7r_0/c$ up to $t = 2r_0/c$, the "collapse" velocity also exceeds the speed of light.

Let us consider a contribution of a circular area of radius $a(t)$ to radiation of the spherical bunch. The geometry of the problem is shown in Fig. 6. The plane xy coincides with the boundary. The circle of radius $a(t)$ represents that layer of the spherical bunch which is situated on the boundary plane at the moment t. Every element of the circle is a source of radiation. Consider a radiation field at the point D at a large distance R_0 from the center of the circle ($R_0 \gg a(t)$). The radius-vector $\mathbf{R}_0$ drawn from the center of the circle to the point D makes an angle θ with the normal to the boundary plane (i.e. with the z-axis). The unit vector $\mathbf{n}$ is defined by $\mathbf{R}_0 = \mathbf{n}R_0$. Without loss of generality we can assume that the point D belongs to the plane xz.

Let us select a small surface element inside the radiating circle, $\mathrm{d}S = r\,\mathrm{d}r\,\mathrm{d}\varphi$, where r is the distance between the surface element and the center of the circle, φ is its azimuth. This element is the source of radiation. Let R be the distance between the radiating element and the point D. The TR field at the point D due to the element $\mathrm{d}S$ is equal to

$$\mathrm{d}E(\omega) = \frac{\sigma}{\pi cR}\frac{\beta\sin\theta}{1 - \beta^2\cos^2\theta}\exp\left(\mathrm{i}\frac{\omega}{c}R\right)\mathrm{d}S, \tag{23}$$

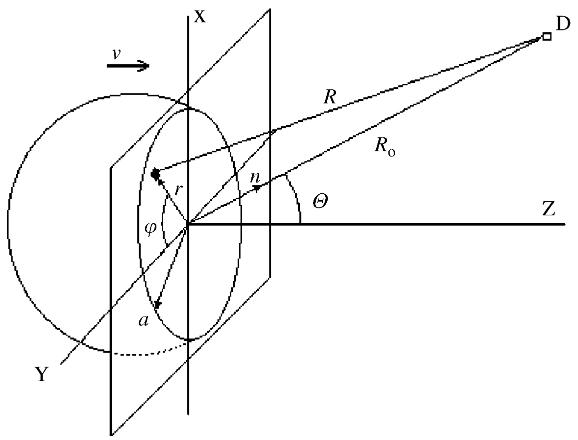

Fig. 6. Geometry for the case of TR generated by a spherical bunch.

where σ is the surface charge density. The field of the radiating area can be obtained by integration over the circle of radius $a(t)$:

$$E(\omega) = \frac{\sigma}{\pi c}\int_0^a r\,\mathrm{d}r\int_0^{2\pi}\frac{\beta\sin\theta}{1-\beta^2\cos^2\theta}\times\frac{1}{R}\exp\left(\mathrm{i}\frac{\omega}{c}R\right)\mathrm{d}\varphi. \quad (24)$$

In our case $R_0 \gg a$, and we may represent the distance R as

$$R = R_0 - \mathbf{nr} = R_0 - r\sin\theta\,\sin\varphi.$$

Taking this relation into account, we get after integration

$$E(\omega) = \frac{2\sigma}{R_0}\frac{\beta\sin\theta}{1-\beta^2\cos^2\theta}\frac{a(t)J_1((\omega/c)\sin\theta\, a(t))}{\omega\sin\theta}. \quad (25)$$

This angular distribution depends on two factors. The first one describes TR emitted by a single point charge, whereas the second factor takes into account interference from different parts of the circular layer.

Radius of the radiating area varies during the passage of the spherical bunch through the surface. The total radiation field $E(\omega)$ can be obtained by integration over all layers or, what is the same, by integration over the time of passage through the boundary. The final expression has the form

$$E(\omega) \propto \frac{2\sigma}{R_0}\frac{\beta\sin\theta}{1-\beta^2\cos^2\theta}\int_0^{2r_0/v}\sqrt{2r_0vt-v^2t^2}\times\frac{J_1((\omega/c)\sin\theta\sqrt{2r_0vt-v^2t^2})}{\omega\sin\theta}\exp\left(\mathrm{i}\frac{\omega}{c}t\right)\mathrm{d}t. \quad (26)$$

Here the integral over t includes an exponent coming from the Fourier transform of the charge distribution of the spherical bunch.

Fig. 7 shows the TR intensity $I(\omega,\theta) \propto E^2(\omega)$ as a function of the radiation angle θ found through Eq. (26) for several ratios of the wavelength λ to the diameter $2r_0$ of the spherical bunch. All shown curves are normalized to the same intensity in the maximum. Note that the sphere is a body which is characterized by only one scale. Two curves 1 in Fig. 7 correspond to the case when the wavelength is larger than the diameter of the bunch: the solid line refers to TR of a bunch and the dotted line gives the intensity for a point-like charge. Difference between the angular distribution for the bunch with $\lambda/2r_0 = 1.5$ and that for the point-like charge is small. Calculations show that the difference becomes even smaller for larger values of the ratio. Curve 2 corresponds to the case $\lambda/2r_0 = 0.73$. In this case radiation of the extended bunch differs considerably from radiation of the point-like charge at large angles. When the ratio further decreases, radiation emitted at small angles becomes more suppressed while radiation at large angles becomes more essential.

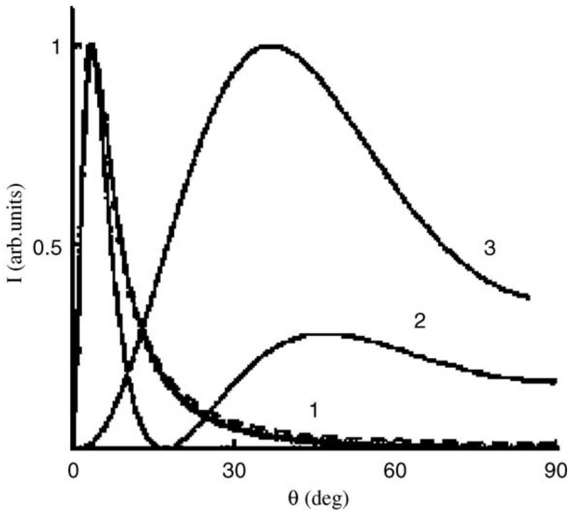

Fig. 7. Angular dependence of the TR intensity for the spherical bunch. Curves 1 (solid and dashed): $\lambda/2r_0 = 1.5$ and $\lambda/2r_0 \gg 1$, respectively. Curve 2: $\lambda/2r_0 = 0.73$. Curve 3: $\lambda/2r_0 = 0.7$.

Curve 3 shows the angular distribution for $\lambda/2r_0 = 0.7$. For such low values of $\lambda/2r_0$ there are additional maxima apart from the narrow maximum at the angle $\theta \simeq 1/\gamma$. Calculations show that the number of maxima increases when the ratio $\lambda/2r_0$ further decreases.

The decrease in radiation in the forward direction can be explained by the fact that in the extended bunch there are volume elements which radiate in counter phase. The decrease becomes especially notable when the length of a bunch becomes comparable with or exceeds the value of the wavelength.

The increase in radiation at large angles can be explained in the following way. During the passage of the bunch through the boundary, the superluminal expansion (and consequent superluminal "collapse") of the radiating spot produces coherent directional radiation. This radiation may be considered as some analog of Vavilov–Cherenkov radiation. Nagorny and Potylitsin (2004) introduced the name "quasi-Cherenkov" for this type of radiation.

The simplest type of transition radiation generated by a relativistic particle that comes out from a conductor perpendicularly to its surface has been investigated in details theoretically and experimentally (Ginzburg and Frank, 1945; Ginzburg and Tsytovich, 1984). As is known, the energy radiated by a particle in the forward direction is zero. Maximum energy is radiated at the angle $\theta_m \simeq 1/\gamma$ towards the velocity direction (here $\gamma = (1-\beta^2)^{-1/2}$ is the relative energy of the particle). As the angle of radiation θ grows after θ_m, the intensity gradually decreases. The energy of radiation emitted along the conductor surface (i.e. under the angles

$\theta \simeq 90^\circ$) is about γ^2 times smaller than that emitted along the direction θ_m.

If the boundary surface is crossed by an extended bunch of charged particles, then transition radiation is the result of interference of radiation emitted by many particles, and total radiation may differ very much from single-particle radiation. This difference becomes especially noticeable for wavelengths comparable with the bunch size. It should be noticed that transition radiation of extended bunches was discussed theoretically in a series of articles (Ginzburg and Tsytovich, 1984). As a rule, their authors were interested in studying conditions, under which TR of a bunch containing N particles would be the same as TR of a single particle of the charge eN. These investigations led to concrete limits imposed on bunch sizes. For practice another problem is also of interest: how do the bunch size and a distribution of particles in the bunch influence angular and spectral distributions of transition radiation? In the case of not a small bunch size the maximum coherence of radiation may not be achieved in the sense that the bunch radiation intensity will not be N^2 times the intensity of single-particle radiation.

In this connection it is of interest and importance to have experimental data on the angular distribution of the TR intensity generated by a real bunch of particles. Recently, measurements of such a distribution were conducted for a bunch of electrons accelerated in a microtron (Serov et al., 2003). These measurements led to an interesting result. It was shown that radiation generated under experimental conditions had properties of both transition radiation and Vavilov–Cherenkov one.

The experimental setup is schematically shown in Fig. 8. A source of relativistic electrons was a microtron operating in the regime of the first type of acceleration. Accelerated electrons had the energy of 7.4 MeV. A pulsed current was 40 mA with the pulse duration of 4 µs. Electrons were extracted from the microtron by means of a magnetic channel, the internal diameter of which was 8 mm. The bunch went into atmosphere through foil 1 located at the microtron flange and having a thickness of 100 µm. Then the bunch crossed foil 2 and produced radiation that was registered by a receiver D, a silicon point diode D-404. The diode was sensitive in the wave range 6–12 mm. The receiver was located in the plane of the microtron orbit (in the plane xz in Fig. 8) at different distances from the bunch axis and could be shifted parallel to this axis.

The second foil size and position against the bunch were subject to change. A copper foil of width 100 µm, length 300 mm and height 200 mm was used in this experiment. To protect the receiver D from radiation generated by the bunch at foil 1, the second foil was bent perpendicularly and located in front of the flange as shown in Fig. 8. The distance d from the bunch axis to the foil bending was 85 mm.

In this experiment the relative energy was $\gamma = E/mc^2 \simeq 15$ and the angle, under which the transition radiation intensity reaches its maximum, was

$$\theta_m = \gamma^{-1} \simeq 3.5^\circ.$$

Angles θ, under which radiation was measured, were much larger than θ_m and fell into the range $45^\circ - 90^\circ$. In accordance with the theory (Ginzburg and Tsytovich, 1984), the radiation intensity for a single charge is proportional to $\sin^2\theta/(1-\beta^2\cos^2\theta)^2$, where $\beta = v/c$ is the ratio of the charge velocity v to the light velocity c. So, the radiation intensity of a single relativistic particle decreases as the angle of radiation grows. For instance,

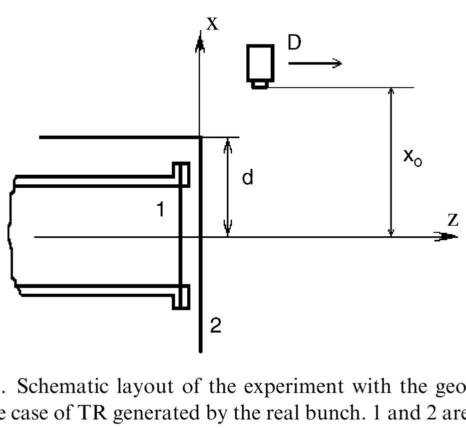

Fig. 8. Schematic layout of the experiment with the geometry for the case of TR generated by the real bunch. 1 and 2 are foils, D is a receiver.

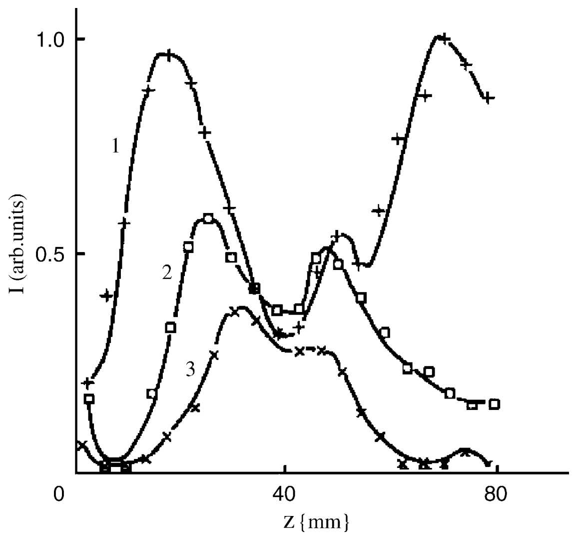

Fig. 9. Measured dependence of the radiation intensity $I(\omega)$ on the longitudinal coordinate z of the detector. $\gamma = 15$, $\lambda \simeq 8$ mm. Curve 1: $x_0 = 95$ mm. Curve 2: $x_0 = 135$ mm. Curve 3: $x_0 = 165$ mm.

the intensity of radiation emitted under the angle 45° is twice as much as that of radiation emitted under 90°.

The experimentally measured dependence of the radiation intensity on the longitudinal coordinate z of the detector D is shown in Fig. 9. The acceptance angle of the radiation receiver was $\simeq 30$ mrad. Since the detector used was not calibrated, the distribution observed experimentally is shown in arbitrary units. Different curves correspond to different values of the transverse coordinate x_0 of the detector D. Measurements were carried out for the wavelength $\lambda \simeq 8$ mm which is comparable with the bunch size. Multiple experiments carried out with the beam from the same microtron have shown that the accelerated bunches had such parameters that the power of coherent radiation even at the wavelength of 400 μm was four to six times bigger than the incoherent radiation power. In the millimeter range, the ratio of coherent and incoherent radiation powers was higher by many orders of magnitude.

As it is seen in Fig. 9, the observed distribution differs very much from that predicted by the theory of transition radiation by a single charge. TR of a single electron has a sharp maximum at the angle $\theta \simeq 1/\gamma$ which is about 3°. In the experiment, transition radiation of a bunch has a sharp maximum at much larger angles. For example, the first maximum on the curve 1 ($x_0 \simeq 95$ mm) is achieved at $z \simeq 20$ mm corresponding to the radiation angle $\theta \simeq 70°$.

The measurements show that there is one more essential difference of transition radiation emitted by a bunch as compared to radiation emitted by a single electron. Namely, the angular distribution of radiation intensity at large angles is azimuthally asymmetric with respect to the z axis. If, for a specified z, one chooses two points, for which x-coordinates are equal in the absolute value but opposite in the sign, then the radiation intensities differ in these two points. In measurements with the same fixed value of z, the radiation intensity at positive values of the transverse coordinate x_0 of the detector was five to eight times higher than that for negative x_0.

The reason for the deviation of the measured angular distribution from that predicted by the theory for a single charge is the coherent nature of radiation. In the experiment, the detector measures radiation which is not emitted by a single electron but is rather a result of interference of waves generated by many electrons in the bunch. This is interference that makes the angular distribution of transition radiation for the bunch so different from that for a single electron. Hereby, the deciding factor that influences the angular distribution is relative positions of radiating particles. In order to explain the experimental data, numerical calculations of the charge distribution inside the bunch accelerated in the microtron and angular distribution of transition radiation generated by this bunch were carried out.

Traditionally, numerical methods are widely used which were developed for a description of particle motion in a microtron (Kapitsa et al., 1961). Experimental studies of the acceleration regime in microtrons showed that these methods describe the processes of electron capture and bunch formation precisely enough. As is known (Kapitsa and Melekhin, 1969), microtron operation depends on several parameters: a size and shape of the accelerating resonator, location of the emitter, amplitudes of the accelerating electromagnetic field and the driving magnetic field. In numerical calculations, therefore, it is necessary to take into account geometric sizes of the resonator and an operation mode of a specific microtron. Results of simulation of particle dynamics in the microtron of Lebedev Institute have been presented by Belovintsev et al. (1981).

Spatial distribution of bunch particles before crossing the foil is shown in Fig. 10. This distribution is obtained via a numerical calculation of electron dynamics in the acceleration mode used in the experiment. It is seen that the bunch has rather sharp boundaries. The boundaries in the transverse direction are determined by the extraction channel diameter and by the interspace length. Form of the bunch in the longitudinal direction is determined by peculiarities of the particle phase motion in the microtron. In the actual operation mode of the microtron bunches had longitudinal size of

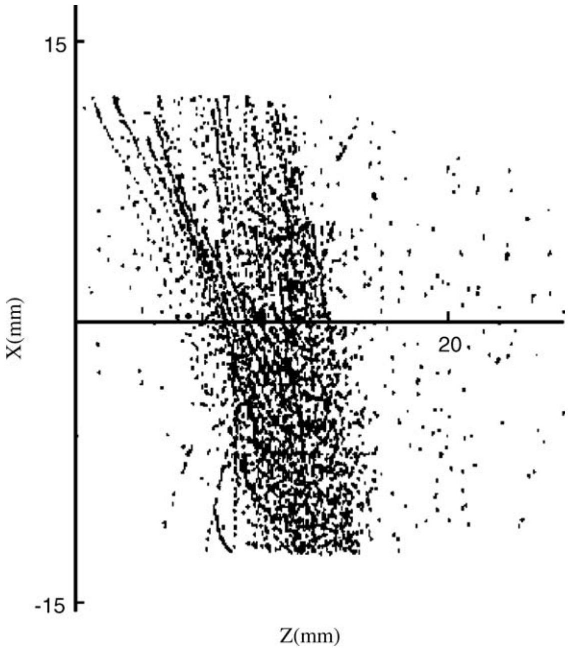

Fig. 10. Spatial distribution of bunch particles before crossing the foil.

$\simeq 9$ mm, horizontal size of $\simeq 28$ mm, and vertical size of $\simeq 4$ mm before the foil.

Using the results of numerical calculations of the spatial particle distribution, transition radiation from a bunch was then calculated. As is known from the theory, if a charged particle crosses a flat metal boundary and comes into vacuum, then the radiation field at frequency ω is described by the expression

$$E(\omega) = \frac{q}{\pi c R} \frac{\beta \sin\theta}{1 - \beta^2 \cos^2} \exp\left(\mathrm{i}\frac{\omega}{c}R - \mathrm{i}\omega t_{\mathrm{i}}\right), \tag{27}$$

where q is the particle electric charge, R is the distance from the transition point to the point of observation, t_{i} is the moment of particle arrival from the metal. The radiation field of the whole bunch is equal to the sum of fields generated by all single particles.

Calculations of the dependence of the transition radiation intensity $I(\omega){\sim}E^2(\omega)$ on the longitudinal coordinate z for a fixed x_0 have been performed for various wavelengths λ. Results of these calculations for the wavelength $\lambda = 8$ mm are shown in Fig. 11. Exactly for that wavelength measurements of the radiation intensity as a function of the longitudinal coordinate z were carried out (see Fig. 9, curve 1). One can see that results of the calculations are qualitatively consistent with the experiment: for $x = 95$ mm the dependence of I on z has a sharp maximum with the amplitude many times larger than that for the case $x = -95$ mm. Still, numerical and experimental results differ somewhat quantitatively: the radiation maximum is attained at the angle $\theta \simeq 60^\circ$ in numerical calculation, whereas this angle is $\theta \simeq 70^\circ$ in the experiment. Possibly, such a difference comes from neglecting, in numerical calculations of the electron motion, some factors that influence the spatial distribution of electrons in the bunch. This indeed may be important because even small changes in the spatial particle distribution within the bunch can essentially influence characteristics of radiation.

The angular dependence of the radiation intensity $I(\omega, \theta)$ in the plane xz is shown in Fig. 12. Calculations have been performed for the case when the distance between the transition point of the bunch and the radiation detector was $R_0 = 100$ mm. As the figure demonstrates, transition radiation of the bunch at large angles with respect to the direction of particle motion is strongly asymmetric. In addition to the maxima at the angles $\theta = \pm 1/\gamma$, there is a maximum of radiation near the angle $\theta \simeq +60^\circ$.

The fact that the bunch front is inclined towards the velocity (i.e. the normal to the bunch front is not parallel to the velocity vector) is important for explanation of these results. The angle between the normal to the front and the velocity is about 10°. So, different parts of the front cross the foil at different time moments. Moreover, the crossing area, that is, the area on the foil from which transition radiation is emitted, changes with time and moves in a transverse direction. It is easy to show that the radiation area on the boundary moves with a superluminal velocity. This superluminal motion of the radiating spot on the boundary surface explains the fact that emitted radiation has features of both transition radiation and, at the same time, Vavilov–Cherenkov radiation. Jointly, all that looks as a source of transition radiation moving perpendicularly to the foil and a source of Vavilov–Cherenkov radiation moving in the plane of the foil. One should bear in mind that every particle of the bunch crossing the foil gives just

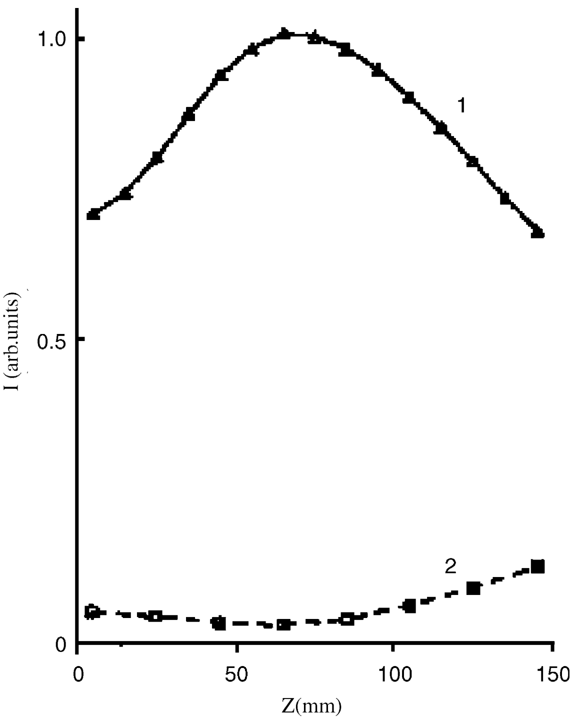

Fig. 11. Numerically calculated radiation intensity $I(\omega)$ versus the longitudinal coordinate z of the detector for $\lambda = 8$ mm. Curve 1: $x_0 = 95$ mm. Curve 2: $x_0 = -95$ mm.

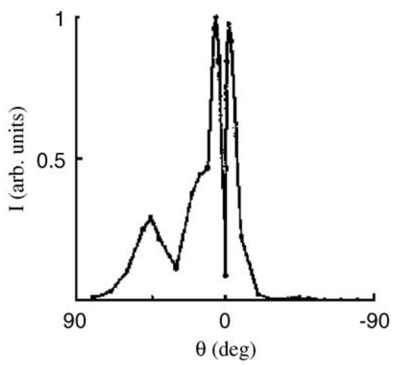

Fig. 12. Numerically calculated angular dependence of the transition radiation intensity at $\lambda = 8$ mm, $R_0 = 100$ mm.

transition radiation but the source of TR moves along the separation surface with a superluminal velocity. As a result of interference of waves emitted by different particles, directional radiation is formed which is characteristic of Vavilov–Cherenkov radiation.

Comparing the experimental data and the classic Vavilov–Cherenkov radiation, one should take experimental conditions into account. If the path L of a superluminal source along the boundary surface is large enough, radiation would have a narrow angular distribution characteristic for traditional sources of Cherenkov radiation. In the experiment, however, the path L of the superluminal source along the foil is only a few (three to four) wavelengths. In such a case the angular distribution of Vavilov–Cherenkov radiation has a finite width $\Delta\theta$ approximately described by the expression

$$\Delta\theta = \frac{\lambda}{2L}\sin\theta_0\cos\theta_0, \tag{28}$$

where θ_0 is the Cherenkov angle. Besides, it should be taken into account that the velocity of the superluminal source in these experiments was not constant because the bunch frontier was not flat and its density was not homogeneous. These conditions also lead to an increase in the angular width. The width decreases with the wavelength decrease.

It should be mentioned that these peculiarities of coherent radiation can be observed in experiments with relativistic beams that have been produced at the majority of linear accelerators. Radiation maxima at large angles will be most sharply expressed in cases when the transverse bunch size is larger than its longitudinal size as it was the case, for example, at the accelerator of the Laboratory of Nuclear Science, Tohoku University (Sendai, Japan) (Takahashi et al., 1994).

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