”Electron-Positron Pair Production and Bresstrahlung at Intermediate Energies in the Field of Heavy Atoms”

Lee, R. N., Milstein, A. I. and Strakhovenko, V. M. (2003), ‘Analytical calculations of the Coulomb corrections to e^+e^- pair production at intermediate photon energies’, hep-ph/0307388, submitted to *Rad. Phys. Chem.*

Lee, R. N., Milstein, A. I. and Strakhovenko, V. M. (2004), ‘High-energy expansion of coulomb corrections to the e^+e^- photoproduction cross section’, *Phys. Rev. A* **69**, 022708.

Lee, R. N., Milstein, A. I., Strakhovenko, V. M. and Schwarz, O. Ya. (2004), ‘Coulomb corrections to bremsstrahlung in electric field of heavy atom at high energies’, hep-ph/0404224, submitted to *Zh. Eks. Teor. Fiz.*

Layout:

I. Introduction: Born approximation, high-energy limit, exact formulae

II. First corrections to the spectrum

1. $O(m/\varepsilon)$-correction
2. Influence of screening
3. Correction to the total cross section of pair production.
4. Comparison with the experimental data.

III. High-energy asymptotic of Coulomb corrections to the differential cross section

1. Influence of screening
2. Finite beam size

IV. Conclusion

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Born approximation *(Bethe and Heitler, 1934)*

Exact in $Z\alpha$, high-energy asymptotics *(Bethe and Maximon, 1954; Davies et al., 1954; Olsen et al., 1957)*

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Extrapolating formula of *Øverbø* (*1977*).

CC to the spectrum

In contrast to the differential cross section, the spectrum of the pair production can be obtained from that of bremsstrahlung and *vice versa*.

$$\frac{d\sigma^{e\to e'\gamma}}{d\omega} = -\frac{\alpha\omega}{2\varepsilon p}\iint d\boldsymbol{r}_1\, d\boldsymbol{r}_2\, \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \sum_{\lambda_\gamma} \mathrm{Sp}\, \left\{\delta G(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon)\,\hat{e}\,\delta G(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon')\,\hat{e}\right\}\ ,$$

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Small angles and large angular momenta $\Rightarrow$ the quasiclassical approximation is applicable.

Within our accuracy one can use the following expression for G

$$G(\boldsymbol{r}_2, \boldsymbol{r}_1|\varepsilon) = \left[\gamma^0(\varepsilon - V(\boldsymbol{r}_2)) - \boldsymbol{\gamma}\cdot\boldsymbol{p}_2 + m\right]$$
$$\times\left[1 + \frac{\boldsymbol{\alpha}\cdot(\boldsymbol{p}_1 + \boldsymbol{p}_2)}{2\varepsilon}\right] D^{(0)}(\boldsymbol{r}_2, \boldsymbol{r}_1|\varepsilon)\,, \quad \boldsymbol{p}_{1,2} = -i\boldsymbol{\nabla}_{1,2}\,,$$

where $D^{(0)}$ is the quasiclassical Green function of the Klein-Gordon equation with the first correction.

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where $D^{(0)}$ is the quasiclassical Green function of the Klein-Gordon equation with the first correction.
First corrections can be represented as a sum of the m/ε correction and the correction due to screening (small in $1/mr_{scr}$).

Correction in m/ε (screening is neglected)

$$D^{(0)}(\boldsymbol{r}_2, \boldsymbol{r}_1|\varepsilon) = \frac{ip\mathrm{e}^{ipr}}{2r_1r_2}\int \frac{d\boldsymbol{q}}{4\pi^2} e^{i\,\frac{prq^2}{2r_1r_2}} \left(\frac{2\sqrt{r_1r_2}}{|\boldsymbol{q}-\boldsymbol{\rho}|}\right)^{2iZ\alpha\lambda} \left(1 + \underbrace{i\frac{\pi(Z\alpha)^2}{2p|\boldsymbol{q}-\boldsymbol{\rho}|}}_{\text{first correction}}\right).$$

(*Lee, Milstein and Strakhovenko, 2000*)

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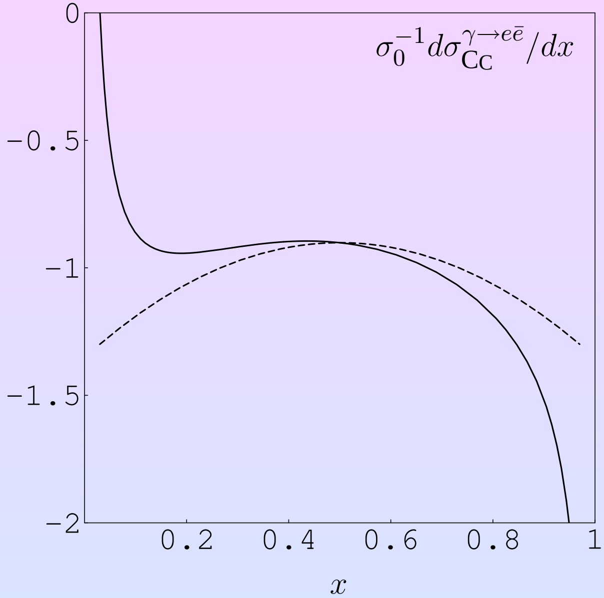
$$y\frac{d\sigma_{\mathrm{Cc}}^{e\to e'\gamma}}{dy} = y\frac{d\sigma_{\mathrm{Cc}}^{e\to e'\gamma(0)}}{dy} + y\frac{d\sigma_{\mathrm{Cc}}^{e\to e'\gamma(1)}}{dy} =$$

$$-4\sigma_0\left[\left(y^2+\frac{4}{3}(1-y)\right)f(Z\alpha) - \frac{\pi^3(2-y)m}{8(1-y)\varepsilon}\left(y^2+\frac{3}{2}(1-y)\right)\mathbf{Re}\, g(Z\alpha)\right]$$

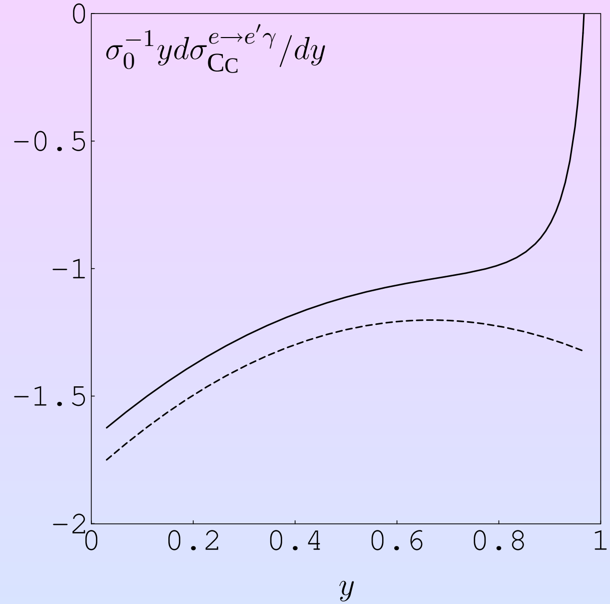
$$y = \omega/\varepsilon\,, \quad \sigma_0 = \alpha(Z\alpha)^2/m^2\,,$$

$$f(Z\alpha) = \mathrm{Re}[\psi(1+iZ\alpha)+C]\,, \quad g(Z\alpha) = Z\alpha\,\frac{\Gamma(1-iZ\alpha)\Gamma(1/2+iZ\alpha)}{\Gamma(1+iZ\alpha)\Gamma(1/2-iZ\alpha)}$$

Spectrum of Cc with $O(m/\varepsilon)$-correction



The dependence of $\sigma_0^{-1} d\sigma_C^{\gamma\to e\bar{e}}/dx$ on x for $Z = 82$, $\omega = 50$ MeV. Dashed curve: leading approximation; solid curve: first correction is taken into account.



The dependence of $\sigma_0^{-1} d\sigma_C^{e\to e'\gamma}/dy$ on y for $Z = 82$, $\varepsilon = 50$ MeV. Dashed curve: leading approximation; solid curve: first correction is taken into account.

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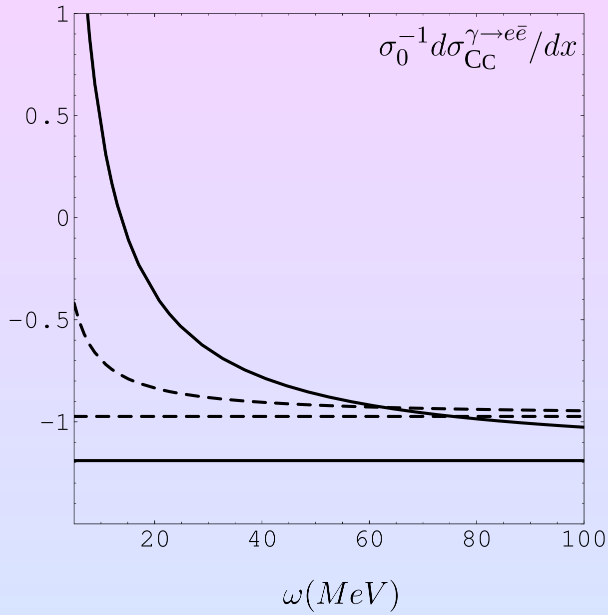
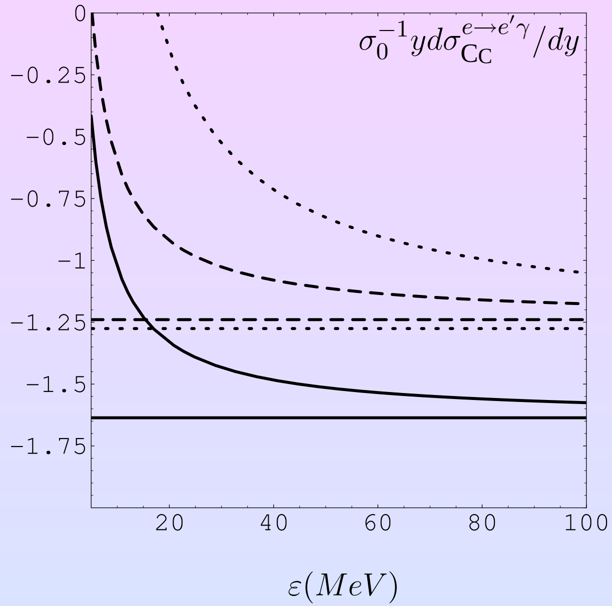


The dependence of $\sigma_0^{-1} d\sigma_{\mathrm{Cc}}^{\gamma\to e\bar{e}}/dx$ on ω for $Z = 82$, $x = 0.1$(solid curve) and $x = 0.3$(dashed curve). Horizontal lines correspond to BM asymptotics.

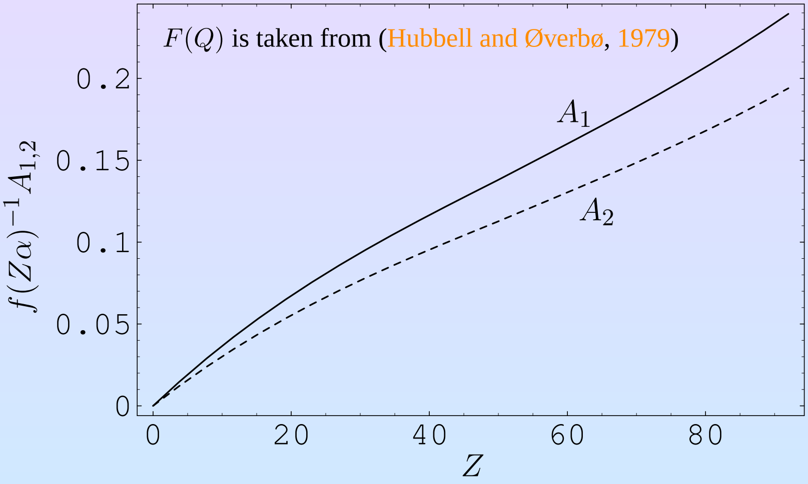


The dependence of $\sigma_0^{-1} y d\sigma_{\mathrm{Cc}}^{e\to e'\gamma}/dy$ on ε for $Z = 82$, $y = 0.1$(solid curve), $y = 0.5$(dashed curve), and $y = 0.9$(dotted curve). Horizontal lines correspond to BM asymptotics.

Screening correction (to leading term in m/ε)

$$D^{(0)}(\boldsymbol{r}_2, \boldsymbol{r}_1|\,\varepsilon) = \frac{ip\mathrm{e}^{ipr}}{8\pi^2 r_1 r_2}\int d\boldsymbol{q}\exp\left[i\frac{pr\,q^2}{2r_1 r_2} - i\lambda r\int_0^1 dxV\left(\boldsymbol{r}_1 + x\boldsymbol{r} - \boldsymbol{q}\right)\right]$$

$$y\frac{d\sigma_{\mathrm{Cc}}^{e\to e'\gamma(scr)}}{\sigma_0 dy} = \left[A_1(1-y) + A_2 y^2\right]\,, \qquad \frac{d\sigma_{\mathrm{Cc}}^{\gamma\to e\bar{e}(scr)}}{\sigma_0 dx} = \left[-A_1 x(1-x) + A_2\right]$$



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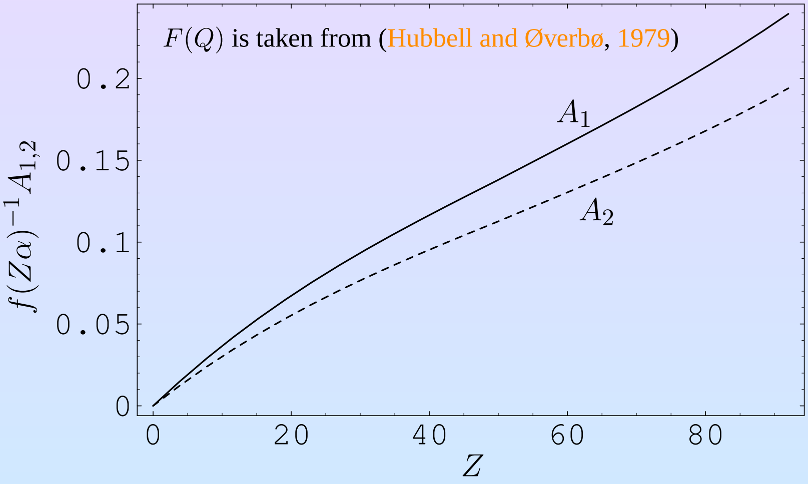

Z-dependence is prompted by Thomas-Fermi and a good fit is

$$A_1\approx 1.2A_2\approx 1.0\cdot 10^{-2}\sigma_0(Z\alpha)^2 Z^{2/3}$$

Corrections to the total cross section of pair production

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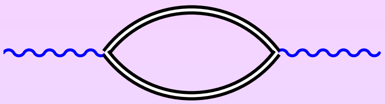
- Correction due to screening can be obtained by trivial integration of the corresponding correction to the spectrum
- $O(m/\varepsilon)$– correction can not be obtained this way

$$\frac{d\sigma_{\mathrm{Cc}}^{\gamma\to e\bar{e}}}{dx} = -4\sigma_0 \left[\left(1 - \frac{4}{3}x(1-x)\right) f(Z\alpha) \right.$$
$$\left. - \frac{\pi^3(1-2x)m}{8x(1-x)\omega} \left(1 - \frac{3}{2}x(1-x)\right) \mathrm{Re}\, g(Z\alpha) \right]$$

The correction is an odd function in $x \to 1-x$ (and $Z\alpha \to -Z\alpha$), therefore does not allow to obtain the correction to the total cross section. The correction to the total cross section comes from the region where one of the particle is not ultrarelativistic $\Rightarrow$ quasiclassical approximation is inapplicable.

Delbrück scattering amplitude.



$$\frac{1}{\omega}\mathrm{Im}\, M^{\gamma\to\gamma} = \sigma^{\gamma\to e\bar{e}} + \sigma^{\gamma\to e(bound)\bar{e}} \,.$$

$$M_{\rm Cc}^{\gamma\to\gamma(0)} + M_{\rm Cc}^{\gamma\to\gamma(1)} = -4i\omega\sigma_0 \int_0^1 dx \left[\left(1-\frac{4}{3}x(1-x)\right) f(Z\alpha) - \frac{\pi^3 m}{8\omega}\left(1-\frac{3}{2}x(1-x)\right)\left(\frac{g^*(Z\alpha)}{x} - \frac{g(Z\alpha)}{1-x}\right)\right].$$

The integral in $M_{\rm Cc}^{(1)}$ is logarithmically divergent. Taking the integral from m/ω to $1-m/\omega$, we find within logarithmic accuracy that Im $M_{\rm Cc}^{(1)}$ vanishes and

$$\mathrm{Re}\, M_{\rm Cc}^{\gamma\to\gamma(1)} = -\frac{\alpha(Z\alpha)^2\pi^3\,\mathrm{Im}\, g(Z\alpha)}{m}\ln\frac{\omega}{m}\,.$$

Dispersion relation

$$\mathrm{Re}M^{\gamma\to\gamma}(\omega) = \frac{2}{\pi}\omega^2\, P\int_0^\infty \frac{\mathrm{Im}M^{\gamma\to\gamma}(\omega')\, d\omega'}{\omega'(\omega'^2-\omega^2)}\,.$$

Large logarithm in Re $M_{\mathrm{Cc}}^{\gamma\to\gamma(1)}$ could appear only if

$$\mathrm{Im}\, M_{\mathrm{Cc}}^{\gamma\to\gamma(1)} = \frac{\alpha(Z\alpha)^2\pi^4\,\mathrm{Im}\, g(Z\alpha)}{2m}\,,$$

at $\omega \gg m$.
The cross section $\sigma^{\gamma\to e(bound)\bar{e}}$ has the form (Milstein and Strakhovenko, 1993)

$$\sigma^{\gamma\to e(bound)\bar{e}} = 4\pi\sigma_0(Z\alpha)^3 f_1(Z\alpha)\frac{m}{\omega}$$

Using these results we obtain

$$\sigma_{\mathrm{Cc}}^{\gamma\to e\bar{e}(1)} = \sigma_0\left[\frac{\pi^4}{2}\mathrm{Im}g(Z\alpha) - 4\pi(Z\alpha)^3 f_1(Z\alpha)\right]\frac{m}{\omega}\,.$$

Estimation of the next correction

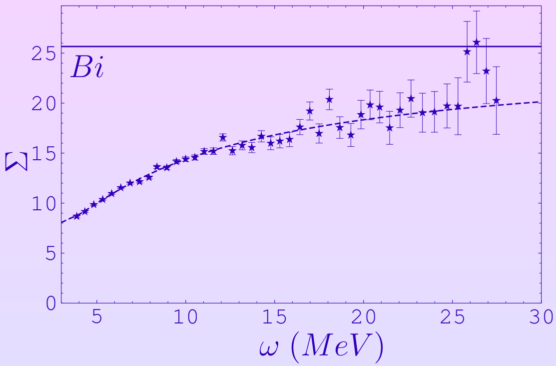
$O(m/\omega)$-correction has large numerical coefficient ($\sim\ 20$ for heavy atoms). How large is the next correction, $O(m^2/\omega^2)$?

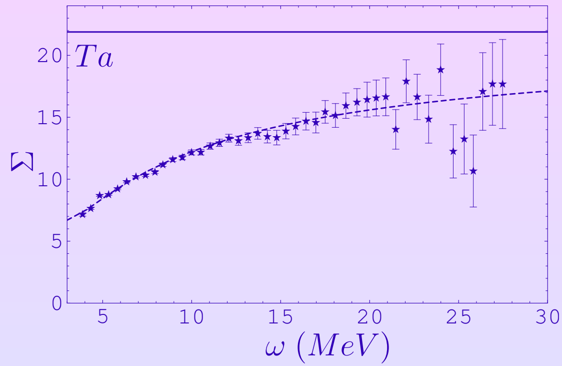
Using the arguments similar to those presented by Davies et al. (1954) the following ansatz for $\sigma_{\mathrm{CC}}^{\gamma\to e\bar{e}(2)}$ has been suggested in our recent paper Lee et al. (2003)

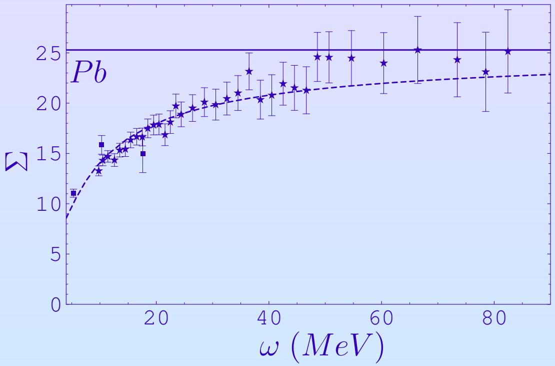
$$\sigma_{\mathrm{CC}}^{\gamma\to e\bar{e}(2)} = \sigma_0 \left[b \ln(\omega/2m) + c\right] \left(\frac{m}{\omega}\right)^2 , \quad \sigma_0 = \alpha(Z\alpha)^2/m^2$$

where b and c are some functions of $Z\alpha$. It was shown in Lee et al. (2003) that experimental data for σ_{coh} are well described if one sets $b = 3.78(\omega/m)\sigma_0^{-1}\sigma_C^{(1)}$, $c = 0$.

Comparison with the experiment



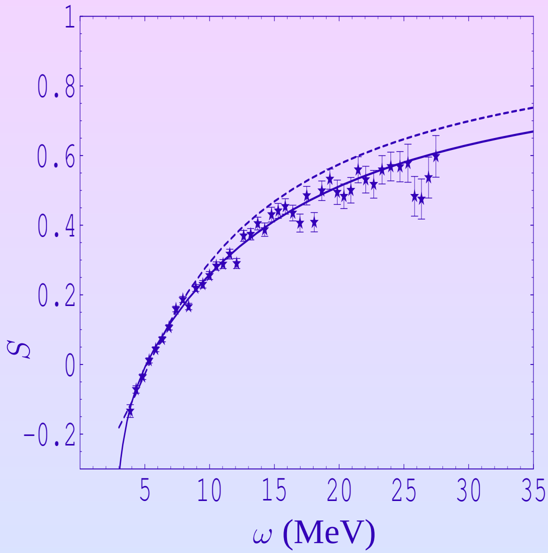




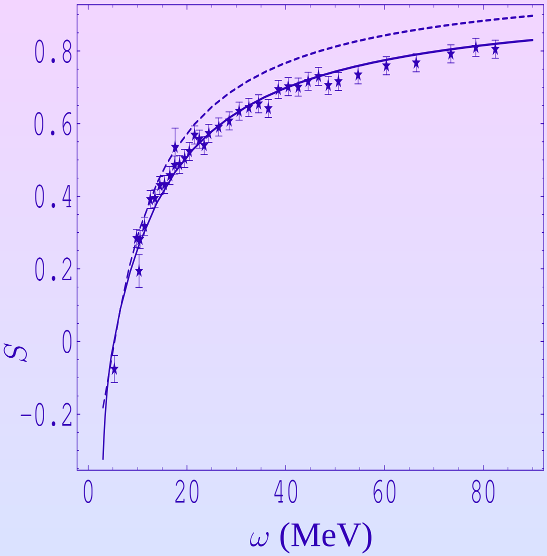
The values of Σ extracted from the experimental data for Bi, Ta, Pb together with the fit $a_{th} + (m/\omega) b \ln(\omega/2m)$ (dashed curve). The solid line represents the asymptotics $\Sigma = a_{th}$,

$$\Sigma = \frac{\omega}{m}\sigma_0^{-1}(\sigma - \sigma_B - \sigma_{\mathrm{CC}}^{(0)} - \sigma_{\mathrm{CC}}^{(scr)}) \,,$$

Comparison with the experiment

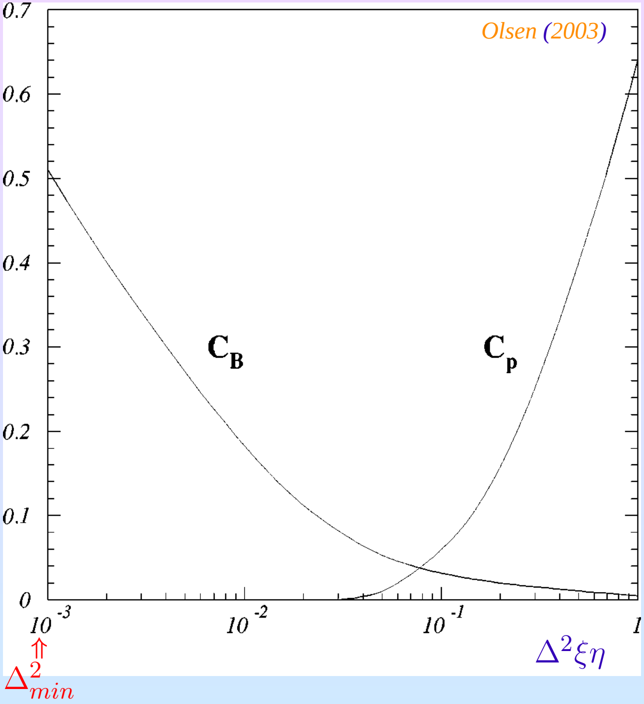


The ω-dependence of $S = (\sigma_{coh} - \sigma_B)/\sigma^{(0)}_{\text{CC}}$ for Bi. Solid curve: our result; dashed curve: the result of Øverbø (1977); experimental data from (Sherman et al., 1980).



Same for Pb.

CC to differential cross section of pair production and bremsstrahlung in the Coulomb field



- CC to pair production cross section come from $\Delta \sim m$
- CC to bremsstrahlung cross section come from $\Delta \sim \Delta_{min}$

CC in the presence of screening

- CC to pair production are insensitive to screening at $r_{scr} \gg 1/m$

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- CC to pair production are insensitive to screening at $r_{scr} \gg 1/m$
- CC to bremsstrahlung at $1/\Delta_{min} \gg r_{scr} \gg 1/m$?

Screening suppresses the region of small momentum transfer therefore a naive answer would be that CC vanish under this condition *(Bethe and Maximon, 1954)*.
However, this can not be true due to Olsen's final-state integration theorem *(Olsen, 1955)*.
Another wrong answer would be that CC to differential cross section of bremsstrahlung are also insensitive to screening *(Olsen, 2003)*.
This is also not true *(Lee et al., 2004)*.

CC to differential cross section of bremsstrahlung in a screened potential

At $\Delta \ll m$ the CC to the cross section has the form

$$d\sigma_C^{e\to e'\gamma} = \frac{\alpha d\omega d\Delta_z d\boldsymbol{\Delta}_\perp}{4\pi^3\varepsilon^2\Delta_z^2}\left[\frac{\varepsilon}{4\varepsilon'} + \frac{\varepsilon'}{4\varepsilon} + \frac{\Delta_{min}}{\Delta_z} + \frac{\Delta_{min}^2}{\Delta_z^2}\right]\frac{dR}{d\boldsymbol{\Delta}_\perp}\,,$$

$$\frac{dR}{d\boldsymbol{\Delta}_\perp} = \left[|\boldsymbol{A}|^2 - |\boldsymbol{A}_B|^2\right]\,,$$

$$\boldsymbol{A}(\boldsymbol{\Delta}) = -i\int d\boldsymbol{r}\,\exp[-i\boldsymbol{\Delta}\cdot\boldsymbol{r} - i\chi(\boldsymbol{\rho})]\boldsymbol{\nabla}_\rho V(\boldsymbol{r})\,,$$

Cc to differential cross section of bremsstrahlung in a screened potential

At $\Delta \ll m$ the Cc to the cross section has the form

$$d\sigma_C^{e\to e'\gamma} = \frac{\alpha d\omega d\Delta_z d\boldsymbol{\Delta}_\perp}{4\pi^3\varepsilon^2\Delta_z^2}\left[\frac{\varepsilon}{4\varepsilon'} + \frac{\varepsilon'}{4\varepsilon} + \frac{\Delta_{min}}{\Delta_z} + \frac{\Delta_{min}^2}{\Delta_z^2}\right]\frac{dR}{d\boldsymbol{\Delta}_\perp}\,,$$

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$$R = \int d\boldsymbol{\Delta}_\perp\int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \mathrm{e}^{-i\boldsymbol{\Delta}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)}\left\{\mathrm{e}^{i\chi(\boldsymbol{\rho}_2)-i\chi(\boldsymbol{\rho}_1)} - 1\right\}$$

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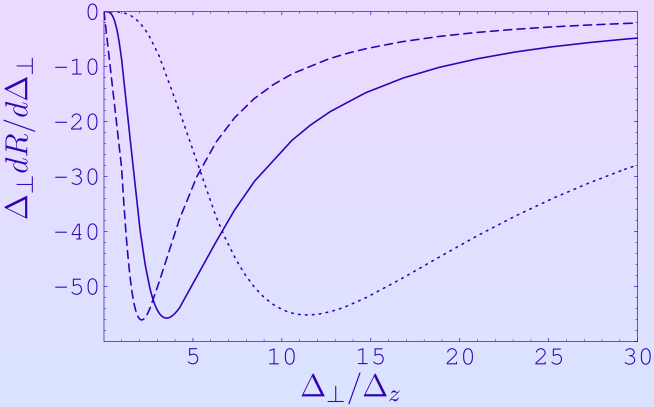
$$R = -32\pi^3(Z\alpha)^2[\mathrm{Re}\psi(1+iZ\alpha)+C] = -32\pi^3(Z\alpha)^2 f(Z\alpha)\,,$$

Cc to differential cross section of bremsstrahlung in a screened potential



Cc to the differential cross section of bremsstrahlung in Ukava potential.
$r_{scr} = \Delta_z^{-1}$(solid curve),
$r_{scr} = 4\Delta_z$(dashed curve),
$r_{scr} = \frac{1}{4}\Delta_z^{-1}$(dotted curve).

The differential cross section is very sensitive to screening. When $r_{scr}^{-1} \gg \Delta_{min}$ the scale of $\Delta_\perp$-distribution is r_{scr}^{-1}. The integral over $\Delta_\perp$ is rapidly converging.

Finite beam size

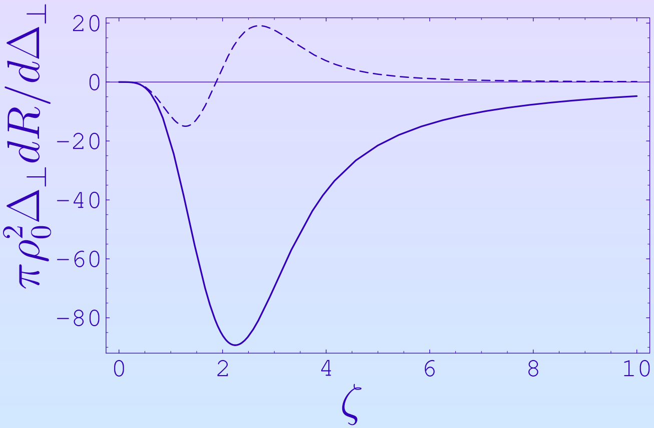
If the peak of the p-distribution has width $\delta p \ll \sqrt{\Delta_{min}\varepsilon} \lesssim m$ then

$$\psi(\boldsymbol{r}) = \phi(\boldsymbol{\rho})\psi^{(in)}_{P_0}(\boldsymbol{r})\,,$$

$\phi(\boldsymbol{\rho})$ has a simple physical meaning: $|\phi(\boldsymbol{\rho})|^2$ is the electrons density (2^D).

$$\boldsymbol{A}(\boldsymbol{\Delta}) = -i\int d\boldsymbol{r}\phi(\boldsymbol{\rho})\exp[-i\boldsymbol{\Delta}\cdot\boldsymbol{r} - i\chi(\boldsymbol{\rho})]\boldsymbol{\nabla}_\rho V(\boldsymbol{r})\,,$$

Again, the differential probability crucially depends on the beam shape.



The quantity $\Delta_\perp\, dR/d\Delta_\perp$ in units $(\pi\rho_0^2)^{-1}$ as a function of $\zeta = \rho_0\Delta_\perp$ for $Z = 80$ and $\phi(\boldsymbol{\rho}) = \phi_0(\rho)$ (solid curve), $\phi(\boldsymbol{\rho}) = \phi_1(\rho)$ (dashed curve).

$$\phi_0(\boldsymbol{\rho}) = \frac{\exp[-\rho^2/2\rho_0^2]}{\sqrt{\pi\rho_0^2}}\,,$$

$$\phi_1(\boldsymbol{\rho}) = \frac{(\rho/\rho_0)^2\exp[-\rho^2/2\rho_0^2]}{\sqrt{2\pi\rho_0^2}}\,,$$

$$\zeta = \rho_0\Delta_\perp\,.$$

$$\frac{dW}{d\omega d\Delta_z} = |\phi(0)|^2\frac{d\sigma}{d\omega d\Delta_z}$$

Results

- The first corrections in m/ε and in $1/mr_{scr}$ to CC in spectrum of bremsstrahlung and pair production are calculated analytically.
- The correction to the total cross section of the pair production is found with the use of the dispersion relation.
- The ansatz for the second correction in m/ε in the total cross section of the pair production is suggested.
- The CC to the differential cross section of bremsstrahlung is shown to crucially depend on screening and on the beam shape.
- The fulfilment of the final-state integration theorem is checked explicitely.

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Total 20slides