Measurements of accelerator beam spectrum by means of Cherenkov radiation intensity dependence on phase velocity of electromagnetic waves in optical and microwaves ranges

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Applications:

-beam energy and energy spectrum control for medical and industrial electron accelerators;

-longitudinal phase space control for high-brightness electron beam.

When compactness is required analyzing magnet at accelerator exit or at some intermediate points of accelerator can not be used.

Requirements:

-non invasive or about non-invasive;

-energy range 3 – 20 MeV;

-energy resolution ~ 1%.

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Background

Theoretical:

Trukhanov K.A. Measurement of particle energy by the dependence of Vavilov - Cherenkov radiation intensity on the phase velocity. In Proc. of the seminar "Cherenkov detectors and their applications in science and techniques" (1984). M. Nauka. 380 –383. 1990

Experimental:

70-MeV and 35-MeV race track microtrons operating at the Skobeltsyn Institute of Nuclear Physics, Moscow State University.

PULSED RACETRACK MICROTRON FOR MAX BEAM ENERGY 70 MeV
1996-2001 with WPT Inc. USA

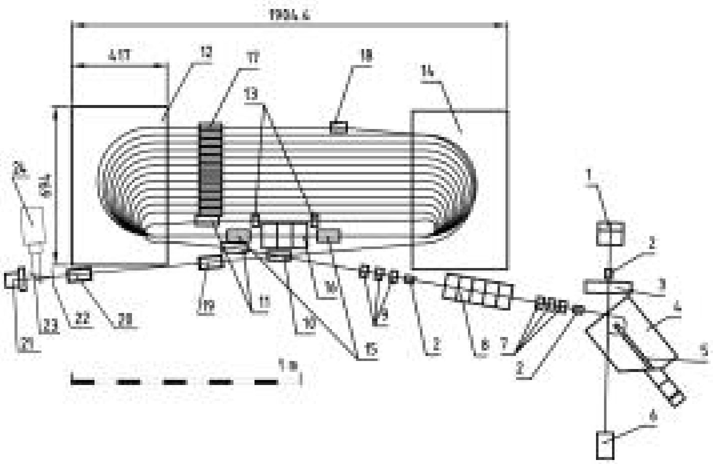
Parameters

Beam energy:	15 – 70 MeV step 5 MeV
Beam current:	2 – 10 mA
Spectrum width	0.2 MeV
Pulse length:	15 µs
Repetition frequency	250 Hz
Klystron	KIU-147A 6 MW/25kW
Operating frequency	2856 MHz
Injection energy	50 keV
Dimensions	0.8*1.4*2.2 m

Features:

Sm-Co as field source in the end magnets.
Accelerating structure with RF quadrupole focusing
Multi-beam klystron with permanent magnet focusing
Self-oscillating mode of klystron operation
Wide use of permanent magnets in optical elements

35 MeV RACETRACK MICROTRON WITH HIGH BRIGHTNESS BEAM
1998-2003 with WPT Inc. USA



Injected beam energy	4.85 MeV
Energy gain per turn	2.43 MeV
Output beam energy	4.85-34.20 MeV
Normalized emittance	10 mm mrad
Longitudinal emittance	200 keV degree
Micro pulse length	5 ps
Pulse repetition rate	1-150 Hz
Micro charge	150 pC
RF frequency	2.856 GHz
Pulsed RF power	< 3 MW
End magnet field	0.486 T

Methods developed for particle detection, e.g. use of Cherenkov radiation cone angle dependence on the particle energy, can not be directly used for accelerator beam energy and energy spectrum control.

Difference with particle detection is in:

- large number of particles and high beam power;
- energy spread in the beam;
- transverse beam dimensions;
- angular spread in the beam.

We consider three methods based on Cherenkov radiation (CR) for accelerator beam energy and energy spectrum control:

1. CR intensity dependence on refraction index in optical range;

2. CR monitor based on gas dispersion in optical range;

3. CR monitors in RF range.

CR intensity dependence on refraction index in optical range near threshold.

$$I(n) \equiv \frac{dJ(n)}{\omega d\omega} = gm \int_{\max\left\{\frac{1}{n},\beta_{min}\right\}}^{\beta_{max}} \left(1 - \frac{1}{n(\omega)^2 \beta^2}\right) f(\beta) d\beta$$

dJ – radiation intensity at frequency ω in interval $d\omega$ for refraction index n

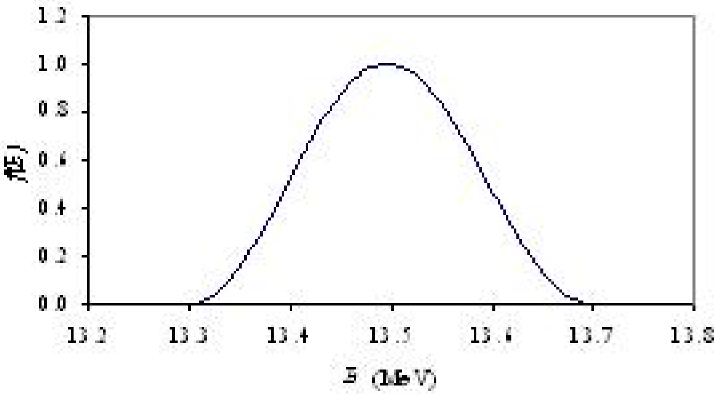
$f(\beta)$ – particle distribution over velocity

β_{min} – minimal velocity in the beam;

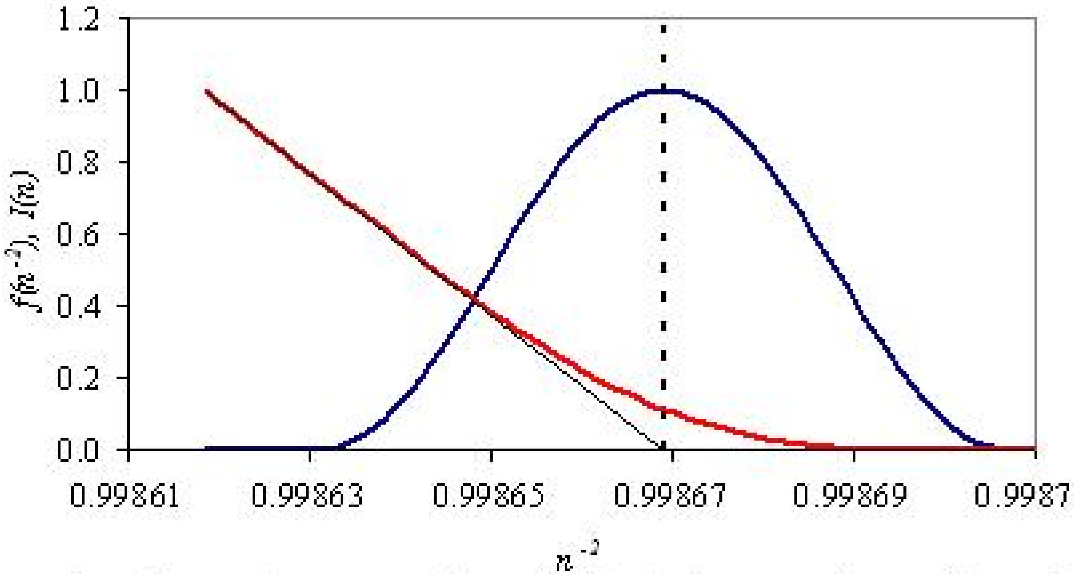
β_{max}- maximum velocity in the beam;

m – radiator mass;

g – some factor.



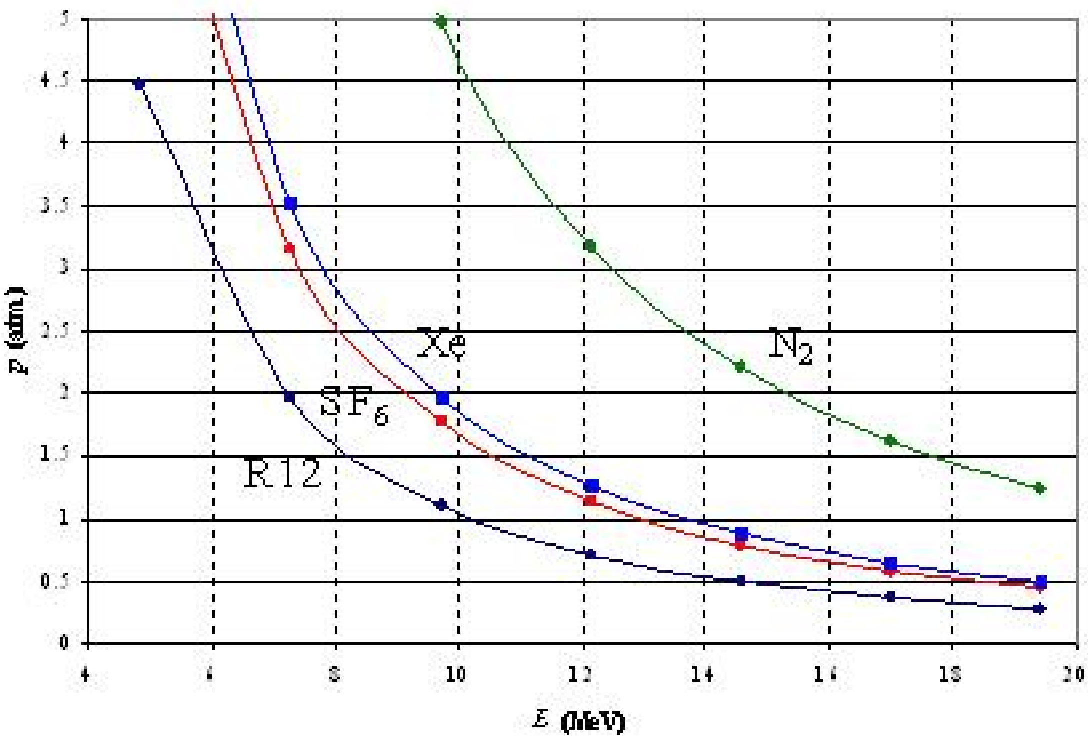
Model energy spectrum



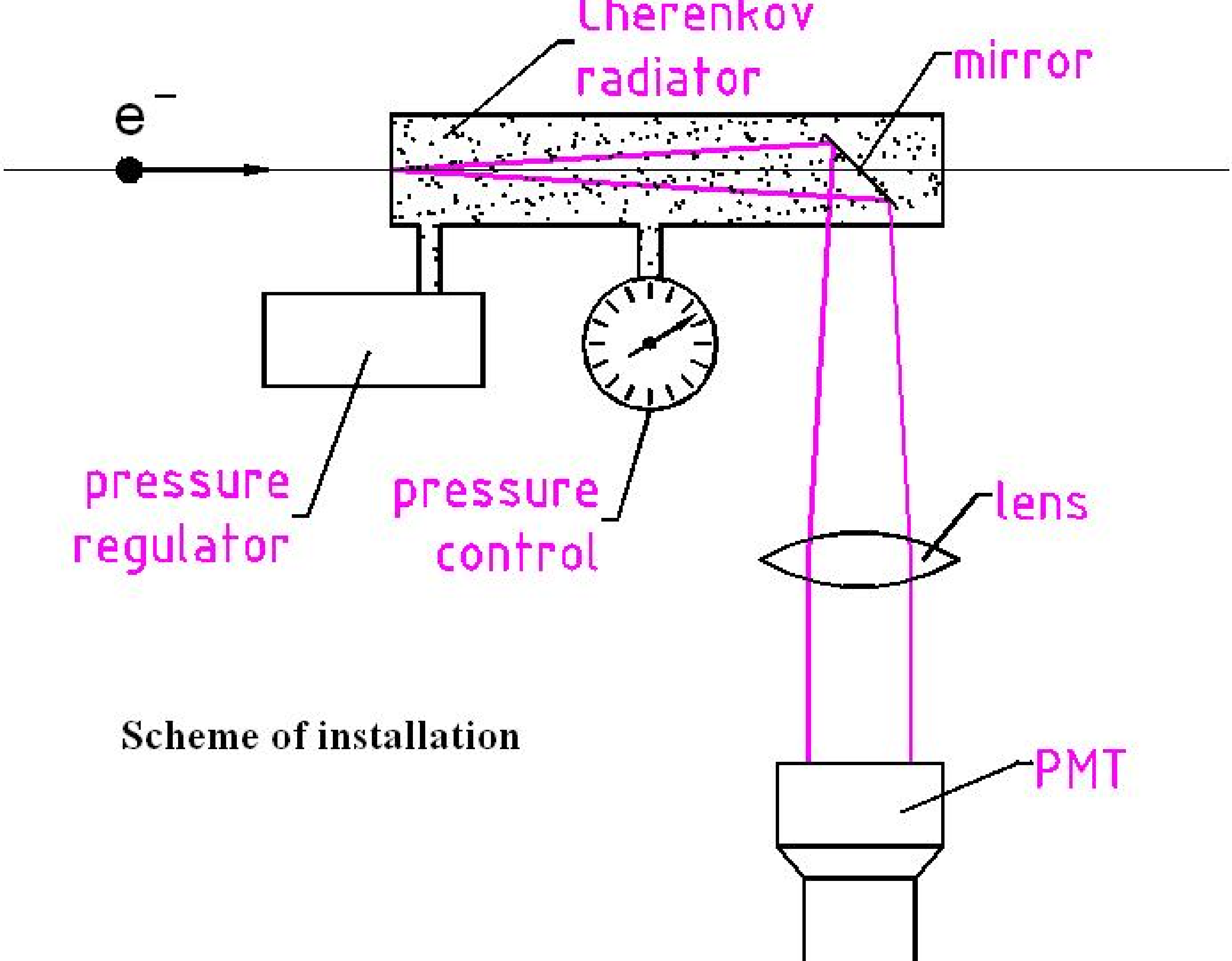
CR intensity dependence on the n^{-2}. Relative number of particles with corresponding to n^{-2} threshold energy are shown.

Near the threshold refraction index for relativistic electrons is very close to 1, so natural choice for Cherenkov radiator material is gas. Refraction index can be varied by varying gas pressure.

$$n(p) = 1 + kp$$



Dependence of the threshold pressure on electron energy for different gases



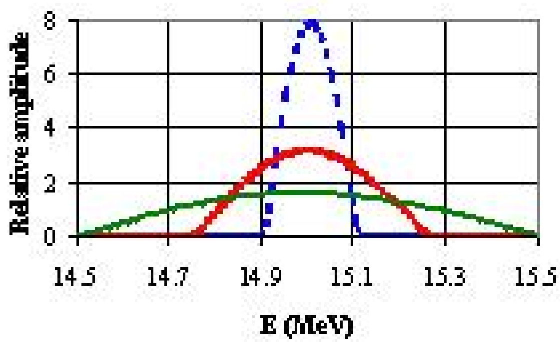
Scheme of installation

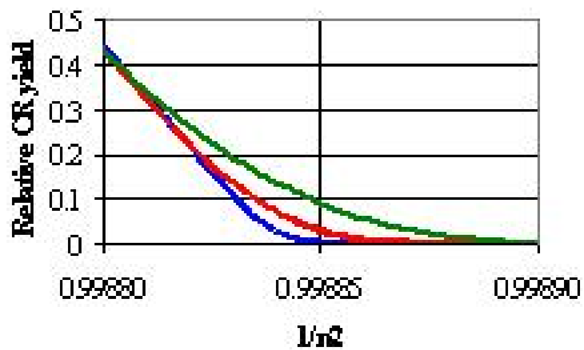
First measurements by this methods of average beam energy were conducted in:

Bhiday M.R., Jennings R.E., Kalmus P.I.P. Measurement of electron beam energy using a gas Cerenkov detector. Proc. Phys. Soc., 72, 973 - 980, 1958

However, not in this work, nor in another work made 15 years later it was mentioned, that nonlinear part of curve contains information about particle velocity distribution.

Trukhanov K.A. Measurement of particle energy by the dependence of Vavilov - Cherenkov radiation intensity on the phase velocity. In Proc. of the seminar "Cherenkov detectors and their applications in science and techniques" (1984).





Energy spectra of different width and their intensity curves near the threshold.

$$gm \int_{\max\left\{\frac{1}{n},\beta_{\min}\right\}}^{\beta_{\max}} \left(1 - \frac{1}{n(\omega)^2 \beta^2}\right) f(\beta)\, d\beta = I(n)$$

This is Volterra integral equation of the first kind with the right part having experimental errors, which is ill-posed task.

$$\beta = y^{1/2}; \quad \Psi(y) = \frac{f(y^{1/2})}{2y^{3/2}}; \quad \frac{1}{n} = z^{1/2}$$

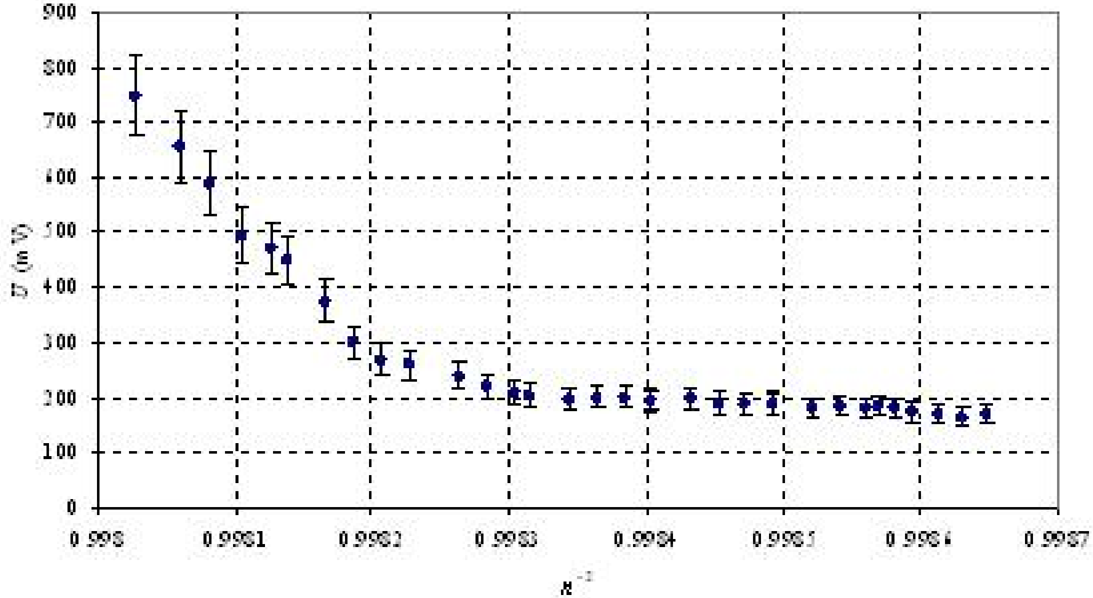
$$gm \int_{z}^{z_{\max}} (y - z)\Psi(y)\, dy = I(z)$$

By successfully differentiating (3) we obtain solution as:

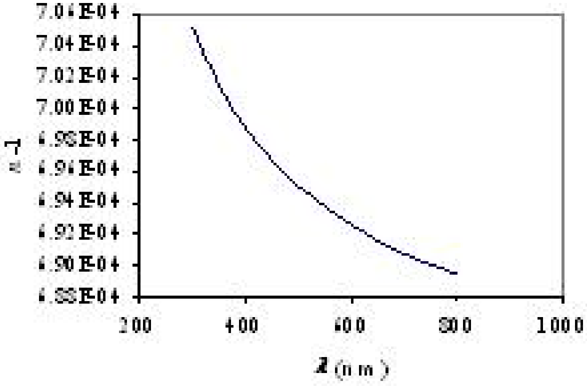
$$\Psi\left(z^{1/2}\right) = \frac{1}{gm} \frac{d^2 I(z)}{dz^2}$$

Which is also ill-posed task. Several methods exist to solve ill-posed tasks, e.g. Tikhonov regularization method, etc.

CR intensity dependence on refraction index measured for beam energy 12.1 MeV with freon R12



CR monitor based on gas dispersion in optical range near threshold

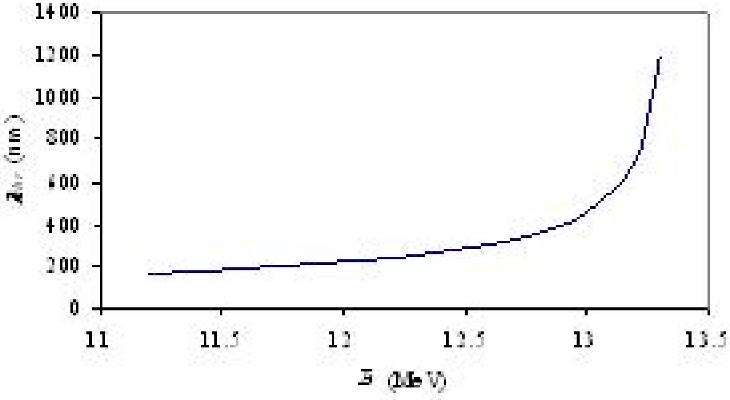


Refraction index dependence on wavelength for Xe , p=1 atm

$$n(\lambda) \approx a + b\lambda^{-2}$$ Cauchy formula for dispersion

$$n(\lambda) = \frac{1}{\beta}$$ Threshold conditions

$$\lambda_{thr}(\beta) = \left(\frac{b}{\beta^{-1} - a} \right)^{1/2}$$ Threshold wavelength



Threshold wavelength dependence on energy for Xe, p=1 atm

The higher particle energy (velocity) for given gas and given pressure the longer wavelengths contribute to photon yield

The number of Cherenkov photons in the wavelengths range $\lambda_2 - \lambda_1$ $(\lambda_2 < \lambda_1)$

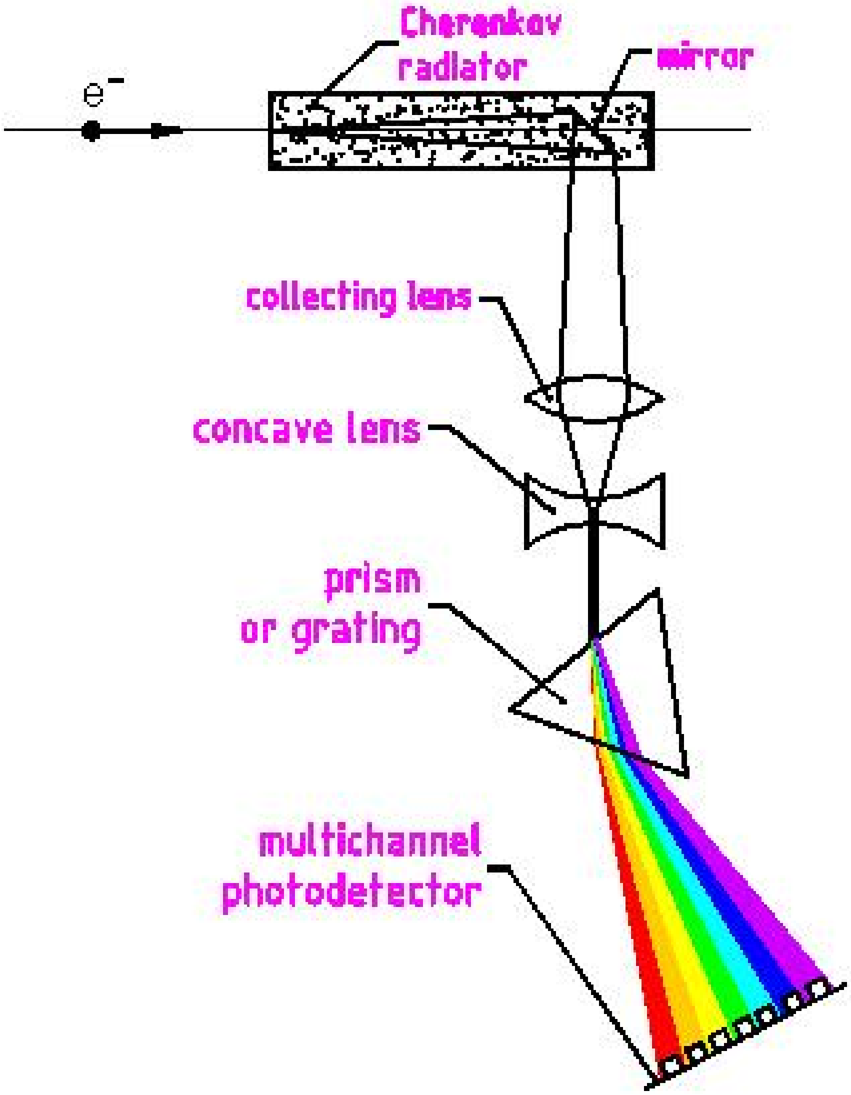
$$N_{ph,\lambda_2-\lambda_1} = 4\pi\alpha N_e k\left[\int_{\beta_{\lambda_1}}^{\beta_{max}} f(\beta)d\beta \int_{\lambda_2}^{\lambda_1}\left(1-\frac{1}{n^2(\lambda)\beta^2}\right)\frac{d\lambda}{\lambda^2} + \int_{\beta_{\lambda_2}}^{\beta_{\lambda_1}} f(\beta)d\beta \int_{\lambda_2}^{\lambda_{th}(\beta)}\left(1-\frac{1}{n^2(\lambda)\beta^2}\right)\frac{d\lambda}{\lambda^2}\right]$$

where α is the fine structure constant, Ne – the number of electrons, k – the photon collection factor

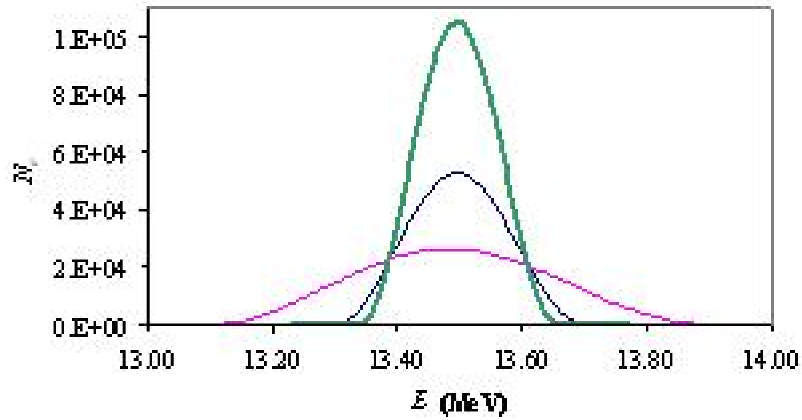
$$f(\beta_{\lambda_2}) = -\frac{1}{4\pi\alpha N_e \frac{dn(\lambda_2)}{d\lambda_2}}\frac{d}{d\lambda_2}\left[\frac{\lambda_2 n^3(\lambda_2)}{\frac{dn(\lambda_2)}{d\lambda_2}}\left(\frac{dN_{ph,\lambda_1-\lambda_2}}{d\lambda_2} + \frac{\lambda_2}{2}\times\frac{d^2N_{ph,\lambda_1-\lambda_2}}{d\lambda_2^2}\right)\right]$$

Particle distribution over velocity can be obtained as combination of first and second derivatives of measured photons yield distribution over wave length. This is also ill-posed task.

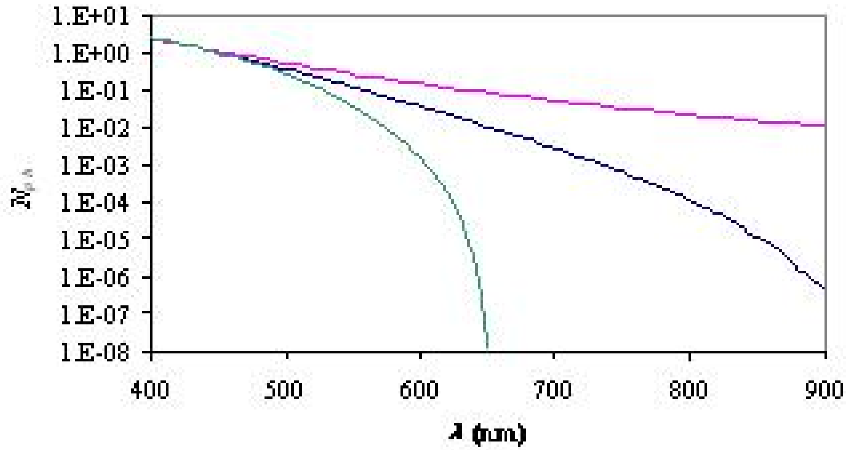
Scheme of installation



Photon spectra corresponding to different electron energy spectra. Xe at pressure 0.95 atm



Electron energy spectra



Photon spectra

Single bunch CR monitors in RF range

Considered monitors are not truly non-invasive. Though entrance/exit windows can be made thin and gas mass thickness is low, beam emittance is essentially deteriorated. During long being in high power beam gas will be dissociated (use of single atomic gas resolve this issue) and heated.

Thus: **having vacuum beam channel in Cherenkov radiator is highly desirable for continuous beam energy and energy spectrum control.**

B.M. Bolotovsky, **The Vavilov-Cherevkov Effect Theory (III)**, Usp. Fiz. Nauk **125** (1961) 295: charge when passing through the vacuum channel with radius b in dielectric radiates in the same manner as in a continuous medium if the next conditions are fulfilled

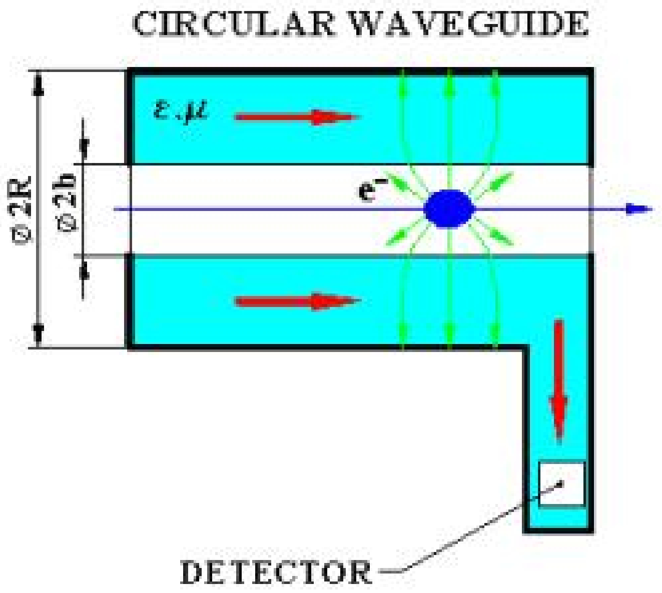
$$b << \frac{\lambda_C \beta}{2\pi\sqrt{1-\beta^2}} \qquad b << \frac{\lambda_C \beta}{2\pi\sqrt{\varepsilon\mu\beta^2 - 1}}$$

Radiation is cut at wavelength:

$$\lambda << \frac{4\pi b\sqrt{1-\beta^2}}{\beta}$$

In practice beam channel radius should not be less than 5 mm, so radiation will take place in mm and cm wavelength ranges.

To provide vacuum at the beam path and to arrange conditions for radiated power registration, dielectric with beam channel should be placed inside conducting metal tube. Similar systems – circular waveguide partially filled by dielectric are studied during about 50 years for electromagnetic field generation and particle acceleration, so results obtained in numerous works can be used for monitor design.



The main feature of the radiation generated by the charge passing through the beam channel in dielectric placed inside conducting tube is that it takes place at discrete frequencies which values are determined by waveguide and beam channel radii, dielectric properties and particle velocity. Charge passing along the beam channel axis will excite with highest amplitude TM_{0n} mode wave with longitudinal electric field on axis. Taking into account conditions above conditions as well as decrease of radiation coherency at the wavelength less than the bunch length, we restrict consideration by lowest TM mode wave. Thus, frequency of excited TM_{01} mode wave and power radiated in this mode are connected with particle velocity:

$$\nu_{01} \approx \frac{\beta c x_{o1}}{2\pi R\sqrt{\varepsilon\mu\beta^2 - 1}}$$

$$p_{o1}(\beta) \approx \frac{2e^2 \beta c}{R^2 \varepsilon\varepsilon_0 J_0'(x_{01})}$$

Strong dependence of the generated radiation frequency on the particle velocity and absence of the sharp boundary for registered signal appearance make it difficult to use method for energy and energy spectrum determination developed for optical wavelength range described in the first part of the report. This problem can be resolved by the use of the high frequency filter cutting off radiation with frequency $\nu > \nu_F$ and by the choice of the ε and μ variation region in accordance with expected range of beam velocities:

$$\frac{1}{\beta_{\max}^2}+\left(\frac{c\chi_{o1}}{2\pi R\,\nu_F}\right)^2 \le \varepsilon\mu \le \frac{1}{\beta_{\min}^2}+\left(\frac{c\chi_{o1}}{2\pi R\,\nu_F}\right)^2$$

For ideal filter with zero attenuation at $\nu \le \nu_F$ and infinite attenuation at $\nu > \nu_F$, registered power will vary with ε or μ variation as:

$$P(\varepsilon\mu)=\int_{\beta_b}^{\beta_{\max}} f(\beta)p_{o1}(\beta)d\beta$$

where

$$\beta_b=\frac{1}{\sqrt{\varepsilon\mu+\left(\dfrac{c\chi_{o1}}{2\pi R\,\nu_F}\right)^2}}$$

Again, this is ill-posed task.

We propose to consider another possibility for beam energy and energy spectrum control using strong dependence of the generated wave oscillation frequency on the particle velocity. Generated wave oscillation frequency is uniquely depended on the particle velocity and for relativistic particles radiated power is nearly independent of the velocity. Thus measurement of the generated radiation spectrum is direct method for beam energy spectrum control not requiring solution of the inverse task. Energy resolution of proposed method is connected with frequency resolution by:

$$\frac{\Delta E}{E} \approx \frac{(\varepsilon\mu\beta^2 - 1)\beta}{1-\beta^2} \times \frac{\Delta \nu}{\nu}$$

Quite simple RF measurement methods, e.g. using high quality factor tunable cavity, provides frequency resolution $\frac{\Delta\nu}{\nu} \approx 10^{-3}$. For beam energy ~ 10 MeV (industrial and medical accelerators) and $\varepsilon\mu \approx 1.1$ (aerogel) energy resolution will be about 4%, and for circular waveguide radius ~ 10 mm radiation will take place in ~8 mm wavelength range.