

Relation between the dipole polarizabilities of charged and neutral pions

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Using the fact that the contribution of the states with isospin $I = 0$ in the difference of the amplitudes of the processes $\gamma\gamma \rightarrow \pi^+\pi^-$ and $\gamma\gamma \rightarrow \pi^0\pi^0$ is very small, we have analyzed the dispersion sum rules for the difference between the dipole polarizabilities of the charged and neutral pions as a function of the σ meson parameters. Then taken into account the current perturbation value of $(\alpha_1 - \beta_1)_{\pi^0} = -1.9$, we have found $(\alpha_1 - \beta_1)_{\pi^\pm} = 9.4 \div 8.2$ for values of the σ meson parameter within the region: $m_\sigma = 400 \div 550$ MeV, $\Gamma_\sigma = 400 \div 600$, $\Gamma_{\sigma \rightarrow \gamma\gamma} = 0 \div 3$ keV. It has been shown that the value of the decay width of the $h_1(1170)$ meson into $\gamma\pi^0$ can be found if the difference $(\alpha_1 - \beta_1)_{\pi^\pm}$ is reliably determined from the experiment. Estimation of the optimal value of the decay width $\sigma \rightarrow \gamma\gamma$ has given $\Gamma_{\sigma \rightarrow \gamma\gamma} \lesssim 0.7$ keV.

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I. INTRODUCTION

Pion polarizabilities are the fundamental structure parameters characterizing the behavior of the pion in an external electromagnetic field. Dipole polarizabilities arise as $\mathcal{O}(\nu_1\nu_2)$ terms in the expansion of the non-Born amplitudes of Compton scattering in powers of the initial and final photon energies ν_1 and ν_2 . In terms of the electric α_1 and magnetic β_1 dipole polarizabilities, the corresponding effective interaction has the form:

$$H_{eff}^{(2)} = -\frac{1}{2} 4\pi (\alpha_1 \vec{E}^2 + \beta_1 \vec{H}^2). \quad (1)$$

The dipole polarizabilities measure the response of the hadron to quasistatic electric and magnetic fields. In what follows, these parameters are given in units 10^{-4}fm^3 .

The values of the pion polarizabilities are very sensitive to the predictions of different theoretical models. Therefore, an accurate experimental determination of them is very important for testing the validity of such models.

At present, the value of the difference of the charged pion dipole polarizabilities found from radiative π^+ meson photoproduction from protons [1] is equal to $11.6 \pm 1.5_{stat} \pm 3.0_{syst} \pm 0.5_{mod}$ and close to the value obtained from scattering of high energy π^- mesons off the Coulomb field of heavy nuclei in Serpukhov [2] and equal to $13.6 \pm 2.8 \pm 2.4$. On the other hand, these values differ from the prediction of the chiral perturbation theory (ChPT) ($4.7 \div 6.7$) [3]. The experiment of the Lebedev Physical Institute on radiative pion photoproduction from protons [4] has given $\alpha_{1\pi^+} = 20 \pm 12$. This value has large error bars but nevertheless shows a large discrepancy with regard to the ChPT predictions, as well.

The preliminary result of the COMPASS collaboration $(\alpha_1 - \beta_1)_{\pi^\pm} = 5.0 \pm 3.4_{stat} \pm 1.2_{syst}$ has been found

by studying the π^- meson scattering off the Coulomb field of heavy nuclei [5]. This result is more close to the ChPT prediction. However to obtain this result the authors used very big values of momentum transfer $Q_{max}^2 \approx 5 \times 10^{-3}$. In this region an interference between the Coulomb and nuclear amplitudes should be taken into account [6–8]. It should be noted that the authors of work [2] chose $Q^2 < 6 \times 10^{-6}$ (GeV/c)² to guarantee that the contribution of the strong interaction below the Coulomb peak is negligible.

The charged pion polarizabilities can be also found by studying the process $\gamma\gamma \rightarrow \pi^+\pi^-$. Investigation of the process $\gamma\gamma \rightarrow \pi\pi$ at low and middle energies was carried out in the framework of different theoretical models and, in particular, in the frame of dispersion relations (DR). In Ref. [9–11] we have analyzed the processes $\gamma\gamma \rightarrow \pi^0\pi^0$ and $\gamma\gamma \rightarrow \pi^+\pi^-$ using DRs with subtractions for the invariant amplitudes M_{++} and M_{+-} without partial-wave expansions. The subtraction constants have been uniquely determined in these works through the pion polarizabilities. The values of the polarizabilities have been found from the fit of the experimental data of the processes $\gamma\gamma \rightarrow \pi^+\pi^-$ and $\gamma\gamma \rightarrow \pi^0\pi^0$ up to 2500 MeV and 2250 MeV, respectively. As a result, we have found $(\alpha_1 - \beta_1)_{\pi^\pm} = 13.0_{1.9}^{+2.6}$ and $(\alpha_1 - \beta_1)_{\pi^0} = -1.6 \pm 2.2$. The result for $(\alpha_1 - \beta_1)_{\pi^\pm}$ is in good agreement with the values obtained in Ref.[1, 2, 4] whereas it is at variance with the ChPT prediction.

In the works [12–15] the dipole polarizabilities of charged pions have been determined from the experimental data of the process $\gamma\gamma \rightarrow \pi^+\pi^-$ in the full energy region $\sqrt{t} < 700$ MeV, (where t is the square of the total energy in $\gamma\gamma$ c.m. system). The results obtained in these works are close to ChPT predictions [3, 16]. However, the values of the experimental cross section of the process $\gamma\gamma \rightarrow \pi^+\pi^-$ in this region [17–20] are very ambiguous, and, as has been shown in Ref. [11, 13], even changes of these values by more than 100% are still compatible with the present error bars.

Therefore, it is necessary to consider other additional possibilities of the $(\alpha_1 - \beta_1)_{\pi^\pm}$ determination.

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Such an information could be obtained from the dispersion sum rules (DSR) for these parameters. However, the main contribution to DSR for $(\alpha_1 - \beta_1)_{\pi^\pm}$ is given by the σ meson, which is very wide and this causes additional uncertainties in the DSR calculation.

On the other hand, if we consider the DSR for the difference between the charged and neutral meson polarizabilities $\Delta((\alpha_1 - \beta_1)) = ((\alpha_1 - \beta_1)_{\pi^\pm} - (\alpha_1 - \beta_1)_{\pi^0})$, then the contribution of mesons with isotopic spin $I = 0$ in the t channel to this difference would be equal 0 [21, 22], when the masses of charged and neutral π mesons are equal each other. As a result, a model dependence for $(\alpha_1 - \beta_1)_{\pi^\pm}$ should be decreased essentially.

In the present work we investigate DSR for $\Delta((\alpha_1 - \beta_1))$ as a function of the decay width of $\sigma \rightarrow \gamma\gamma$, when the masses of the charged and neutral π mesons are not equal each other. As will be shown, the contribution of the σ meson is small in this case too, and we can find a realistic limit on the value of $(\alpha_1 - \beta_1)_{\pi^\pm}$.

It has been shown that the value of the decay width of the $h_1(1170)$ meson into $\gamma\pi^0$ can be found if the value $\Delta((\alpha_1 - \beta_1))$ is reliably determined from experiment.

II. DISPERSION SUM RULES FOR THE PION POLARIZABILITIES

We will consider the helicity amplitudes M_{++} and M_{+-} . These amplitudes have no kinematical singularities or zeros [23]. The relations between the amplitudes $\gamma\gamma \rightarrow \pi^+\pi^-$, $\gamma\gamma \rightarrow \pi^0\pi^0$ and the ones with isotopic spins $I = 0$ and $I = 2$ read

$$\begin{aligned} F_C &= \sqrt{\frac{2}{3}} \left(F^0 + \frac{1}{\sqrt{2}} F^2 \right), \\ F_N &= \sqrt{\frac{2}{3}} \left(F^0 - \sqrt{2} F^2 \right). \end{aligned} \quad (2)$$

The dipole (α_1 and β_1) polarizabilities are defined [10, 24] through expansion of the non-Born helicity amplitudes of Compton scattering on the pion in powers of t at fixed $s = \mu^2$

$$\begin{aligned} M_{++}(s = \mu^2, t) &= 2\pi\mu(\alpha_1 - \beta_1) + \mathcal{O}(t), \\ M_{+-}(s = \mu^2, t) &= 2\frac{\pi}{\mu}(\alpha_1 + \beta_1) + \mathcal{O}(t), \end{aligned} \quad (3)$$

where μ^2 is the π meson mass (different for π^0 and π^\pm), $t + s + u = 2\mu^2$.

The dispersion sum rules for the difference of the dipole polarizabilities was obtained in Ref. [9] using DRs at fixed $u = \mu^2$ without subtractions for the amplitude M_{++} . In this case, the Regge-pole model allows the use

of DR without subtractions [23]. Such a DSR is

$$\begin{aligned} (\alpha_1 - \beta_1) &= \frac{1}{2\pi^2\mu} \left\{ \int_{4\mu^2}^{\infty} \frac{ImM_{++}(t', u = \mu^2) dt'}{t'} \right. \\ &\quad \left. + \int_{4\mu^2}^{\infty} \frac{ImM_{++}(s', u = \mu^2) ds'}{s' - \mu^2} \right\}. \end{aligned} \quad (4)$$

As is evident from Eq.(2), the contribution of the isoscalar mesons to the difference $\Delta((\alpha_1 - \beta_1))$ equals 0 (if the masses of the charged and neutral pions are equal). We will study this difference when these masses do not equal each other.

The DSRs for the charged pions are saturated by the contributions of the $\rho(770)$, $b_1(1235)$, $a_1(1260)$, and $a_2(1320)$ mesons in the s -channel and σ , $f_0(980)$, $f_0'(1370)$ in the t -channel. For the π^0 meson the contribution of the ρ , $\omega(782)$, ϕ , $h_1(1170)$, and $b_1(1235)$ mesons are considered in the s -channel and the same mesons as for the charged pions in the t -channel. Besides, we take into account a nonresonant S-wave contribution of two charged pions in the t channel.

The parameters of the ρ , ω , ϕ , b_1 and a_2 mesons are given by the Particle Data Group [25]. For the $a_1(1260)$ meson we took $m_{a_1} = 1230$ MeV [25], $\Gamma_{a_1} = 425$ MeV (the average value of the PDG estimate [25]), $\Gamma_{a_1 \rightarrow \gamma\pi^\pm} = 0.64$ MeV [26].

The parameters of the $f_0(980)$ and $f_0'(1370)$ mesons are taken as follows:

$f_0(980)$: $m_{f_0} = 980$ MeV [25], $\Gamma_{f_0} = 70$ MeV (the average of the PDG [25] estimate), $\Gamma_{f_0 \rightarrow \gamma\gamma} = 0.56 \times 10^{-3}$ MeV, $\Gamma_{f_0 \rightarrow \pi\pi} = 0.84 \Gamma_{f_0}$ [27], $\Gamma_{f_0 \rightarrow K\bar{K}} = 0.16 \Gamma_{f_0}$;
 $f_0'(1370)$: $m_{f_0'(1370)} = 1430$ MeV, $\Gamma_{f_0'(1370)} = 145$ MeV, $\Gamma_{f_0'(1370) \rightarrow \gamma\gamma} = 0.54 \times 10^{-5}$ MeV [28], $\Gamma_{f_0'(1370) \rightarrow \pi\pi} = 0.26 \Gamma_{f_0'(1370)}$ [29].

The mass and the total decay width of the $h_1(1170)$ meson are taken from PDG: $m_{h_1} = 1170$ MeV, $\Gamma_{h_1} = 360$ MeV. The decay $h_1 \rightarrow \gamma\pi^0$ has not yet been observed. Therefore we use this decay width according the work [15]:

$$\Gamma_{h_1 \rightarrow \gamma\pi^0} = \frac{e^2}{4\pi} C_{h_1} \frac{(m_{h_1}^2 - m_{\pi^0}^2)^3}{3m_{h_1}^3}, \quad (5)$$

where the coefficient C_{h_1} can be estimated using nonet symmetry [15, 30]:

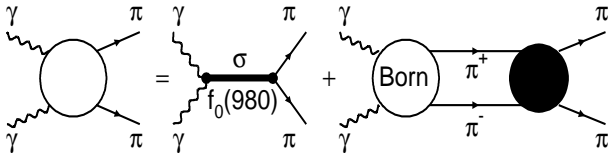
$$C_{h_1(1170)} \simeq 9C_{b_1(1235)} \simeq 0.45. \quad (6)$$

As a result we have $\Gamma_{h_1 \rightarrow \gamma\pi^0} \simeq 1.6845 \pm 0.44$ MeV.

Recently, a lot of works have been devoted to the study of the σ meson (see, for example [31–35]). An average of the most advanced data on the σ meson gives ([35])

$$m_\sigma = 446 \pm 6, \quad \Gamma_\sigma/2 = 276 \pm 5. \quad (7)$$

In our analysis we use the values of the mass of the σ meson and its total decay width in the following intervals:

FIG. 1: S -wave contributions

$$m_\sigma = 425 \div 550 \text{ MeV}, \quad \Gamma_\sigma = 400 \div 600 \text{ MeV}.$$

The values of the decay width of $\sigma \rightarrow \gamma\gamma$ we consider from 0 up to 3 keV.

Expressions for the imaginary parts of the resonances under consideration are given in Appendix.

Besides the contribution of the σ , $f_0(980)$, and $f_0^1(1370)$ mesons we have taken into account a non-resonant contribution of the S -waves with the isospin $I = 0$ and 2 according to the diagrams of Fig. 1.

It is worth noting that the vertexes of the σ and f_0 meson poles in the dispersion approach include the full dynamics of the transitions on the mass shell. In this case there is no need to consider direct and rescattering mechanisms of transition separately.

According to the unitarity condition, the imaginary part of the amplitude M_{++} for the $\pi^+\pi^-$ -loop diagram in Fig. 1 can be written as

$$\text{Im}M_{++}^{(s)} = \mathcal{B} \text{Re}T_{\pi^+\pi^- \rightarrow \pi\pi}, \quad (8)$$

where $\mathcal{B} \equiv \mathcal{B}(\gamma\gamma \rightarrow \pi^+\pi^-)$ is the contribution of the Born amplitude to the S -wave of the $\gamma\gamma \rightarrow \pi^+\pi^-$ amplitude and equal to

$$\mathcal{B} = 16\pi \left(\frac{e^2}{4\pi} \right) \frac{m_{\pi^\pm}^2}{t^2} \ln \left(\frac{1 + q/q_0}{1 - q/q_0} \right), \quad (9)$$

$q(q_0)$ is the momentum (energy) of the π meson. The Born amplitude can be expressed in terms of the $I = 0$ and $I = 2$ isospin amplitudes as

$$\mathcal{B} = \sqrt{\frac{2}{3}}B^{(I=0)} + \sqrt{\frac{1}{3}}B^{(I=2)}. \quad (10)$$

Taking into account that

$$\mathcal{B}(\gamma\gamma \rightarrow \pi^0\pi^0) = -\sqrt{\frac{1}{3}}B^{(I=0)} + \sqrt{\frac{2}{3}}B^{(I=2)} = 0 \quad (11)$$

we have [36]

$$B^{(I=0)} = \sqrt{\frac{2}{3}}\mathcal{B}, \quad B^{(I=2)} = \sqrt{\frac{1}{3}}\mathcal{B}. \quad (12)$$

The amplitudes of $\pi\pi$ scattering are expressed through the amplitudes in the isotopic space $T^{(I=0)}$ and $T^{(2)}$ as follows:

$$\begin{aligned} T_{\pi^+\pi^- \rightarrow \pi^+\pi^-} &= \frac{2}{3} \left(T^{(0)} + \frac{1}{2}T^{(2)} \right), \\ T_{\pi^+\pi^- \rightarrow \pi^0\pi^0} &= \frac{2}{3} \left(T^{(0)} - T^{(2)} \right). \end{aligned} \quad (13)$$

B_0	7.26 ± 0.23	d_0	$(227.1 \pm 1.3)^\circ$	ϵ_1	4.7 ± 0.2
B_1	-25.3 ± 0.5	B	$(94.0 \pm 2.3)^\circ$	ϵ_2	-15.0 ± 0.8
B_2	-33.1 ± 1.2	C	$(40.4 \pm 2.9)^\circ$	ϵ_3	4.7 ± 2.6
B_3	-26.6 ± 2.3	D	$(-86.9 \pm 4.0)^\circ$	ϵ_4	0.38 ± 0.34

TABLE I: The values of the coefficients in Eqs. (17,19). [37]

According to the relations (10) and (13) the imaginary parts of the $\pi\pi$ loop contributions to the S -wave of the amplitude M_{++} are equal to

$$\text{Im}M_{++}^{(s)}(\gamma\gamma \rightarrow \pi^0\pi^0) = \frac{4}{9}\mathcal{B} \text{Re} \left(T^{(0)} + T^{(2)} \right), \quad (14)$$

$$\text{Im}M_{++}^{(s)}(\gamma\gamma \rightarrow \pi^+\pi^-) = \frac{1}{9}\mathcal{B} \text{Re} \left(4T^{(0)} + T^{(2)} \right). \quad (15)$$

The amplitudes $T^{(0)}$ and $T^{(2)}$ can be presented as

$$\text{Re}T^{(I)} = \frac{q_0}{q}\eta_I \cos\delta_0^I(t) \sin\delta_0^I(t), \quad (16)$$

where $\delta_0^I(t)$ is the phase-shift of the S -wave of $\pi\pi$ scattering with isospin I and η_I is the inelasticity.

The expression for the phase-shift $\delta_0(t)$ has been determined using the parameterization of Ref. [37]. At low energy $t \lesssim 4m_k^2$ we have

$$\begin{aligned} \cot\delta_0^0(t) &= \frac{\sqrt{t}}{2q} \frac{\mu^2}{t - \frac{1}{2}\mu^2} \left\{ \frac{\mu}{\sqrt{t}} + B_0 + B_1w(t) \right. \\ &\quad \left. + B_2w(t)^2 + B_3w(t)^3 \right\}, \end{aligned} \quad (17)$$

where

$$w(t) = \frac{\sqrt{t} - \sqrt{4m_k^2 - t}}{\sqrt{t} + \sqrt{4m_k^2 - t}},$$

and η_0^0 is equal to 1.

For the energy $4m_k^2 < t < (1.42 \text{ GeV})^2$ we use [37]

$$\delta_0^0(t) = d_0 + B \frac{q_k^2}{m_k^2} + C \frac{q_k^4}{m_k^4} + D\theta(t - 4m_\eta^2) \frac{q_\eta^2}{m_\eta^2}, \quad (18)$$

$$\begin{aligned} \eta_0^0(t) &= \exp \left[\frac{-q_k}{\sqrt{t}} \left(\epsilon_1 + \epsilon_2 \frac{q_k}{\sqrt{t}} + \epsilon_3 \frac{q_k^2}{t} \right)^2 \right. \\ &\quad \left. - \epsilon_4 \theta(t - 4m_\eta^2) \frac{q_\eta}{\sqrt{t}} \right], \end{aligned} \quad (19)$$

where $q_k = \sqrt{t/4 - m_k^2}$ and $q_\eta = \sqrt{t/4 - m_\eta^2}$; m_k and m_η are the masses of K and η mesons, respectively.

The parameters in Eqs(17-19) are listed in Table 1.

To describe the phase-shift $\delta_0^2(t)$ we use Schenk's parameterization [38] in the energy region up to 1.5 GeV, assuming that $\eta_0^2 = 1$ [39]

$$\tan\delta_0^2 = \frac{q}{q_0} \left\{ A_0^2 + B_0^2 q^2 + C_0^2 q^4 + D_0^2 q^6 \right\} \left(\frac{4\mu^2 - s_0^2}{t - s_0^2} \right), \quad (20)$$

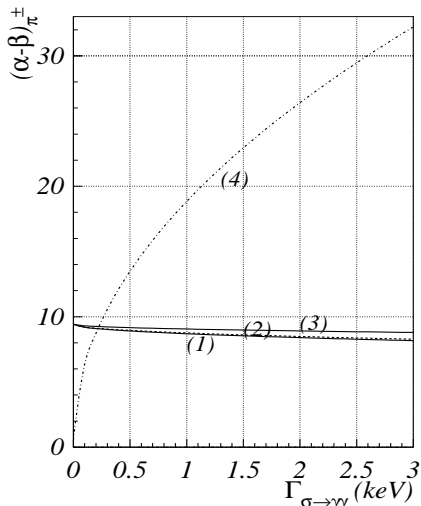


FIG. 2: Dependence of $(\alpha_1 - \beta_1)_{\pi^\pm}$ on $\Gamma_{\sigma \rightarrow \gamma\gamma}$. Lines (1), (2), and (3) correspond to the calculation of $\Delta((\alpha_1 - \beta_1))$ at $\sqrt{t_\sigma} = 446 - i276$, $400 - i300$, and $550 - i200$ MeV, respectively. Line (4) is the result of the calculations of DSR (4) for $(\alpha_1 - \beta_1)_{\pi^\pm}$ at $\sqrt{t_\sigma} = 446 - i276$

where

$$A_0^2 = -0.044, B_0^2 = -0.0855\mu^2, C_0^2 = -0.00754\mu^4, \\ D_0^2 = 0.000199\mu^6, s_0^2 = -11.9\mu^2.$$

III. CALCULATION OF $(\alpha_1 - \beta_1)_{\pi^\pm}$

The value of $(\alpha_1 - \beta_1)_{\pi^0}$ has been determined from the investigation of the process $\gamma\gamma \rightarrow \pi^0\pi^0$ in the works [9, 14, 15]: -1.6 ± 2.2 , -0.6 ± 1.8 , $-1.25 \pm 0.08 \pm 0.15$.

These values are in good agreement with the prediction of ChPT [40] $(\alpha_1 - \beta_1)_{\pi^0}^{ChPT} = -1.9 \pm 0.2$. Therefore, in order to determine $(\alpha_1 - \beta_1)_{\pi^\pm}$ we have added the value of $(\alpha_1 - \beta_1)_{\pi^0}^{ChPT}$ to the results of the calculations of $\Delta((\alpha_1 - \beta_1))$ with help of DSR (4) at different values of the decay width of $\sigma \rightarrow \gamma\gamma$, when the mass and the total decay width of the σ meson vary within the following values: $m_\sigma = 400 \div 550$ MeV, $\Gamma_\sigma = 400 \div 600$ MeV.

The results of the calculation are shown in Fig.2. Line (1) corresponds to calculations with $\sqrt{t_\sigma} = m_\sigma - i1/2\Gamma_\sigma = 446 - i276$. Lines (2) and (3) correspond to $m_\sigma = 400$ MeV, $\Gamma_\sigma = 600$ MeV and $m_\sigma = 550$ MeV, $\Gamma_\sigma = 400$ MeV, respectively. As is evident from this Figure the values of $(\alpha_1 - \beta_1)_{\pi^\pm}$ weakly depend on the mass and the total decay width of the σ meson in the region under consideration. The values of $(\alpha_1 - \beta_1)_{\pi^\pm}$ obtained are within $9.4 \div 8.2$.

The greatest contribution to $\Delta((\alpha_1 - \beta_1))$ is given by the ω and h_1 mesons. The parameters of the ω meson and so its contribution are well known. On the other hand

the experimental data on the h_1 meson are very poor. In particular, the decay of this meson into $\gamma\pi^0$ was not observed yet still in the experiment. Therefore, a reliable experimental determination of $\Delta((\alpha_1 - \beta_1))$ will allow to determine the real value of the decay width $\Gamma_{h_1 \rightarrow \gamma\pi^0}$. For example, if the result of work [1] $(\alpha_1 - \beta_1)_{\pi^\pm} = 11.6$ is confirmed, then $\Gamma_{h_1 \rightarrow \gamma\pi^0} = 0.875$ MeV.

Line (4) in Fig.1 is the result of the calculations of DSR (4) for $(\alpha_1 - \beta_1)_{\pi^\pm}$ at $m_\sigma = 446$ MeV and $\Gamma_\sigma = 552$ MeV. This result strongly depends on the decay width $\Gamma_{\sigma \rightarrow \gamma\gamma}$ and indicates that realistic values of $(\alpha_1 - \beta_1)_{\pi^\pm}$ can be obtained if $\Gamma_{\sigma \rightarrow \gamma\gamma} \lesssim 0.7$ keV.

The influence of the upper integration limit (Λ) in the DSR (4) on the results of the calculation was investigated. They are not practically changed for Λ more than $(6 \text{ GeV})^2$. In the present work we performed the integration up to $(20 \text{ GeV})^2$.

IV. CONCLUSIONS

Using the fact that the contribution of the state with isospin $I = 0$ to the difference $\Delta((\alpha_1 - \beta_1)) = (\alpha_1 - \beta_1)_{\pi^\pm} - (\alpha_1 - \beta_1)_{\pi^0}$ is very small we have analyzed DSR for this difference at the real values of the pion masses. DSR has been calculated for the σ meson parameters within the intervals: $m_\sigma = 400 \div 550$ MeV, $\Gamma_\sigma = 400 \div 600$, $\Gamma_{\sigma \rightarrow \gamma\gamma} = 0 \div 3$ keV. In order to determine $(\alpha_1 - \beta_1)_{\pi^\pm}$ we have added $(\alpha_1 - \beta_1)_{\pi^0}^{ChPT} = -1.9$ to $\Delta((\alpha_1 - \beta_1))$. The values of $(\alpha_1 - \beta_1)_{\pi^\pm}$ found weakly depend on the σ meson parameters and are in the range $(\alpha_1 - \beta_1)_{\pi^\pm} = 9.4 \div 8.2$. This result is in agreement with the experimental values obtained in work [1], whereas it is at variance with the calculations in the framework of ChPT [3].

It has been shown that further experimental investigation of $\Delta((\alpha_1 - \beta_1))$ can be an opportunity to determine the decay width $\Gamma_{h_1 \rightarrow \gamma\pi^0}$.

Besides, the analysis of DSR for $(\alpha_1 - \beta_1)_{\pi^\pm}$ showed that more realistic values of this parameter ($(\alpha_1 - \beta_1)_{\pi^\pm} < 15$) can be obtained with help of DSR (4) if the decay width $\Gamma_{\sigma \rightarrow \gamma\gamma} \lesssim 0.7$ keV. The values $\Gamma_{\sigma \rightarrow \gamma\gamma} \lesssim 1$ keV were obtained early in works [9, 10, 41, 42] also. Results with $\Gamma_{\sigma \rightarrow \gamma\gamma} > 1$ keV quoted in the resent literature are listed in [33, 43]

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Appendix A

The contributions of the vector and axial-vector mesons (ρ, ω, ϕ, a_1 , and b_1) to $ImM_{++}(s, u = \mu^2)$ are calculated with the help of the expression

$$ImM_{++}^{(V)}(s, u = \mu^2) = \mp 4g_V^2 s \frac{\Gamma_0}{(m_V^2 - s)^2 + \Gamma_0^2} \quad (A1)$$

where m_V is the meson mass, the sign "+" corresponds to the contribution of the a_1 and b_1 mesons and

$$g_V^2 = 6\pi \sqrt{\frac{m_V^2}{s}} \left(\frac{m_V}{m_V^2 - \mu^2} \right)^3 \Gamma_{V \rightarrow \gamma\pi} D_1(m_V^2)/D_1(s), \quad (A2)$$

$$\Gamma_0 = \left(\frac{q_i^2(s)}{q_i^2(m_V^2)} \right)^{\frac{3}{2}} \frac{m_V^2}{\sqrt{s}} D_1(m_V^2)/D_1(s) \Gamma_V \quad (A3)$$

Here D_1 is connected with the centrifugal potential and equal to $D_1 = 1 + (q_i r)^2$ [44], $r = 1\text{fm}$ is an effective interaction radius, Γ_V and $\Gamma_{V \rightarrow \gamma\pi}$ are the total decay width and the decay width into $\gamma\pi$ of these mesons. The momentums q_i^2 for (ρ, ω, ϕ, a_1 , and b_1) mesons are equal to $(s - 4\mu^2)/4$, $(s - 9\mu^2)/4$, $(s - 4m_k^2)/4$, $(s - (m_\rho + \mu^2)/4)$, and $(s - 16\mu^2)/4$, respectively.

Appendix B

The amplitude of the contribution of a scalar meson to the process $\gamma\gamma \rightarrow \pi\pi$ can be written as

$$T = \frac{g_s}{\sqrt{t} - M_s - i\frac{1}{2}\Gamma_s}. \quad (B1)$$

Then it is easy to show that the imaginary part of the amplitude $ImM_{++}^\sigma(t)$ of the σ meson contributions to the process under consideration could be presented as

$$ImM_{++}^\sigma(t) = \frac{g_\sigma(\sqrt{t} + M_\sigma)\Gamma_0^\sigma(t)}{(t - M_\sigma^2)^2 + (\Gamma_0^\sigma(t))^2}, \quad (B2)$$

where

$$g_\sigma = \frac{8\pi}{t} \left[\frac{2}{3} \frac{M_\sigma \Gamma_{\gamma\gamma} \Gamma_\sigma}{\sqrt{M_\sigma^2 - 4\mu^2}} \right], \quad (B3)$$

$$\Gamma_0^\sigma = \frac{M_\sigma(\sqrt{t} + M_\sigma)}{2\sqrt{t}} \left(\frac{t - 4\mu^2}{M_\sigma^2 - 4\mu^2} \right)^{1/2} \Gamma_\sigma. \quad (B4)$$

These expressions (B2)-(B4) can be very useful to describe scalar mesons with large decay widths.

As the two K mesons give a big contribution to the decay width of the $f_0(980)$ meson and the threshold of the reaction $\gamma\gamma \rightarrow K\bar{K}$ is very close to the mass of the $f_0(980)$ meson, we consider Flatté's expression [45] for the $f_0(980)$ meson contribution to the process $\gamma\gamma \rightarrow \pi\pi$.

For $t > 4m_k^2$:

$$ImM_{++}^{f_0} = g_{f_0} \frac{\Gamma_{0f_0}}{(m_{f_0}^2 - t)^2 + \Gamma_{0f_0}^2}, \quad (B5)$$

where

$$\Gamma_{0f_0} = \left[\Gamma_{f_0 \rightarrow \pi\pi} \left(\frac{t - 4\mu^2}{m_{f_0}^2 - 4\mu^2} \right)^{1/2} + \Gamma_{f_0 \rightarrow kk} \left(\frac{t - 4m_k^2}{m_{f_0}^2 - 4m_k^2} \right)^{1/2} \right] m_{f_0}. \quad (B6)$$

For $t < 4m_k^2$:

$$ImM_{++} = g_{f_0} \Gamma_{0f_0} \left([m_{f_0}^2 - t - \left(\frac{4m_k^2 - t}{m_{f_0}^2 - 4m_k^2} \right)^{1/2} m_{f_0} \Gamma_{f_0 \rightarrow kk}]^2 + \Gamma_{0f_0}^2 \right)^{-1} \quad (B7)$$

$$\Gamma_{0f_0} = \Gamma_{f_0 \rightarrow \pi\pi} m_{f_0} \left(\frac{t - 4\mu^2}{m_{f_0}^2 - 4\mu^2} \right)^{1/2}. \quad (B8)$$

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