Peculiarities of the nucleus-internal target interaction at the Nuclotron

Alexander S. Artiomov
Laboratory of High Energies, Joint Institute for Nuclear Research, 141980 Dubna, Russian Federation

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Abstract

Peculiarities of the nucleus-internal target interaction at synchrotrons are theoretically investigated. Taking this interaction into account, analytical expressions for luminosities averaged over cycle time, time evolution of the current, transverse and longitudinal emittances are obtained. Graphical functions, characterizing the luminosities and parameter evolution of an d, C and Ar nucleus beam with energies of 1 and 6 A GeV for different internal targets at the Nuclotron are presented.

1. Introduction

Relativistic nuclear physics deals with the study of processes in which the constituents of nuclear matter move with relative velocities close to the velocity of light. The asymptotic character of such natural phenomena has played a decisive role in the construction of the Nuclotron, a strong focusing superconducting accelerator of relativistic nuclei at the Laboratory of High Energies of the Joint Institute for Nuclear Research in Dubna [1]. The Nuclotron provides an experimental investigation of nucleus-nucleus interactions for beam energies from 5 MeV to 6 GeV per nucleon. Using internal targets and a circulating beam with an intensity of \( \sim 5 \times 10^6 \) deuterons per spill, physical experiments are being performed over a beam energy range of 100 MeV-2.3 GeV per nucleon [2,3]. This energy range is characterized by transition conditions from proton-neutron to quark–gluon matter [4]. The planning of internal target experiments requires a detailed knowledge of the ion-target interaction. This interaction is sufficiently well studied for light ion beams circulating in synchrotrons (see, for example, Refs. [5-9]). As shown in this paper, for more heavy ions the ion-internal target interaction has some peculiarities. Taking these into account, the luminosities averaged over cycle time and parameter evolution of a d, C and Ar nucleus beam with energies of 1 and 6 A GeV for different internal targets at the Nuclotron are investigated.

2. Beam-internal-target interaction at the synchrotrons and evolution of ion beam parameters

Physics experiments with internal targets are usually realized in recirculation mode of synchrotron operation after beam injection and acceleration. It is important for this mode of operation that the mean energy loss of ions per target traversal is compensated by an appropriate synchrotron acceleration in the rf-cavity. If we suppose that ions traverse a homogeneous target every turn and residual gas effects are negligible, general analytical expressions for beam parameter evolution can be obtained.

Small angle scattering and energy loss straggling of ions lead to growth of transverse and longitudinal beam emittances. Using the Courant–Snyder formalism [10] for transverse phase ellipses and taking an initial Gaussian distribution of ions over \( (r',X) \) coordinates, the following correlation can be obtained

\[
\langle (\tilde{Y})^2 \rangle = 2\beta \varepsilon^{(1)}(\eta)/\eta^2.
\]

Here \( \langle \cdot \rangle \) is the mean value over the all ions in a beam, \( Y = X, Z \) are the horizontal (radial) and vertical (axial) displacements from the equilibrium orbit, respectively; \( Y' \) is the trajectory slope, \( (\tilde{Y})^2 = Y^2 + (\alpha Y + \beta Y')^2 \); \( \pi \varepsilon^{(1)}(\eta) \) is the ellipse area before ion-internal target interaction corresponding to \( \eta \) standard deviations in the distribution and enclosing a \( \xi \) part of all ions in the beam \( (\eta = 1 \rightarrow \xi = 0.384, \eta = 2 \rightarrow \xi = 0.865, \eta = 3 \rightarrow \xi = 0.9891) \); \( \beta, \alpha, D_i, D'_i, \gamma_i = (1 + \alpha_i^2)/\beta, \) (see below) are the synchrotron parameters at the target location. When a fast ion survives the \( j \)-traversal of the internal target, the change \( \delta Y'_j \) of the trajectory slope and the relative momentum deviation \( \delta_i = \Delta p_i/p \) take place. The deviation \( \delta Y'_j \) caused by the target can be neglected. Assuming the number of betatron oscillations per turn \( Q_i = m/n, \) \( (m, \text{and} \ n, \text{are integers}) \), \( \mu = 2\pi Q_i \) and using Twiss's transport matrices (see, for example, Ref. [11]), we obtain the deviations...
after an "elementary act" of ion-internal target interaction (effect of \( n_i \) target traversals and following turns) in the absence of oscillation damping. The \( \delta Y_i(n_i) \) and \( \delta Y_i'(n_i) \) distributions with the mean values of \( \delta Y_i(n_i) = \delta Y_i'(n_i) = 0 \) and their mean square deviations are independent of the "elementary act" number. Thus, from the fundamental limit theorem of probability theory, we can assume that the resulting \((Y', Y)\)-distributions after many "elementary acts" of ion-internal target interaction will also be Gaussian. From this and using correlation (1), the following expression on the evolution of transverse beam emittances \( \epsilon_i'(n) \) can be obtained

\[
\epsilon_i'(n) = \epsilon_i^{(0)}(n) + 0.5N\beta_i\eta^2(\delta Y')^2 + 0.5N\eta^2\left[ \sum_{j=1}^{n_i-1} (D_j + D'_j) + \sum_{j=1}^{n_i-1} \left| D'_j \right| \sin \mu_j \right] \delta_{j+1},
\]

where \( N \) is the number of target traversals, \( (\delta Y')^2 \) and \( \overline{\delta}^2 \) are the mean square deviations in angle and relative momentum after target traversal; \( \eta = \eta_x, \eta_z \).

For small synchrotron oscillation amplitudes with Gaussian distribution in longitudinal \((\Delta \varphi, \delta)\)-phase space \((\Delta \varphi\) is the rf-phase lag with respect to the \( \varphi \)-phase of the synchrotron particle) and from formal analogy between upright longitudinal and similar transverse \((\varphi = 0)\) phase ellipses, one can obtain the following equation for the evolution of the longitudinal emittance

\[
\epsilon_i'(n) = \epsilon_i^{(0)}(n) + 0.5N\beta_i\eta^2(\delta Y')^2.
\]

Fig. 1. The \( f_1 \) (solid line) and \( f_2 \) (dashed line) functions vs target mass number \( A_0 \) for different incident nuclei.
of \((\delta Y)^2\) values are small since the values of \(\theta_m\) and \(\theta_{cm}\) enter logarithmically into Eq. (6).

The growth of beam emittances, inelastic nuclear scattering and large angle elastic scattering in a single projectile passage through the target lead to beam losses. If we take into account only the first channel, the time evolution for circulating beam intensity (in relative units) can be obtained from the Fokker-Planck model of projectile diffusion in the \((X', Y')\)-phase space [11]. As the longitudinal emittance growth can be influenced by the rf-cavity voltage (see Eq. (5)), we suppose that particle losses in the longitudinal phase space can be made negligible. Using the results of Ref. [14], the probability of projectile loss because of diffusion in a beam after \(N\) target traversals is estimated by

\[
P(N) = \prod_{i=1}^{N} P(\epsilon^{(N)}_t),
\]

where calculated \(P(\epsilon^{(N)}_t)\)-function is shown in Fig. 2. Approximating this function by \(\exp[-5.36(\epsilon_t - 0.06)]\), the effective cross section \(\sigma_t\) of projectile loss after start time \(\tau_t\) can be obtained as

\[
\sigma_t \approx \frac{A_0}{A_t}\left[ \frac{\beta_t f(\tau)}{1 + f_1(\tau)} \right]^{1/2} \frac{25}{\tau_t^2 Bc} \left[ \frac{\beta_t f_1(\tau)}{1 + f_2(\tau)} \right]^{-1/2} \equiv \frac{S}{t \beta c} \alpha(\tau), \tag{8}
\]

where \(S, A_t\) are the ring circumference and acceptance.

Inelastic nuclear interaction and large angle elastic scattering lead to the projectile loss in every passage through the target with the cross section

\[
\sigma_{\text{loss}} = \sigma_t \equiv \frac{A_0 S}{t \beta c} \left( 6 \times 10^{-26} (A_t^{1/3} + A_0^{1/3})^2 \right), \sigma_\text{in} \approx 3 \times 10^{-3} \left[ 2.0Z/(\beta^2\gamma A) \right]^2
\]

and Gaussian approximation of a central maximum of plane diffractional nuclear scattering is used. In Eq. (10) we should assume \(\theta_{cm} = \theta_{cm} \) when \(\theta_{cm} > \theta_{cm}\). Depending on the collision energy and the type of colliding nuclei, different terms dominate in \(\sigma_{\text{loss}}\). The average beam lifetime \(T_b\) and cross section of the projectile loss \(\sigma_{\text{loss}}\) can be estimated as

\[
T_b \approx a + \sum_{i=1}^{\infty} \tau_f b_i, \quad (i = x, z) \tag{11}
\]

\[
\sigma_{\text{loss}} = \sigma_t + \sigma_{\text{el}} = a/T_b, \tag{12}
\]

where \(a = A_0 S/(6 \times 10^{24} t \beta c), b_i = 1\) if \(\tau_i < T = a/\sigma_t \equiv I(1/s/\tau)\) and \(b_i = 0\) otherwise, \(\sigma_{\text{el}}\) is the effective cross section of the diffusion projectile loss.

The luminosity is the product of beam current and target thickness averaged over time. In recirculation mode of the accelerator run the luminosity \(L_c\) averaged over the cycle time \(T_c\) has the maximum value of \(L_c = N_0/(T_c \sigma_{\text{loss}})\) \((N_0\) is the number of the circulating particles before beam-target interaction) which is independent of target thickness when \(T \geq T_c = \frac{A_0 S}{(6 \times 10^{24} t \beta c)}, L_c = \frac{N_0}{(6 \times 10^{24} t \beta c) \sigma_{\text{loss}}}\). For internal target thickness \(T < T_c\) the luminosity is decreased by the value of \(I/T_c\), where \(S, A_t\) are the ring circumference and acceptance.

Inelastic nuclear interaction and large angle elastic scattering lead to the projectile loss in every passage through the target with the cross section

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3. Internal target effects at the Nuclotron

Using the above results, numerical calculations of internal target effects for d, C and Ar nuclei with energies of 1 and 6 A GeV are obtained at the Nuclotron (S = 250 m, A = 40 mm mrad and ε(ν) ≈ 2 mm mrad for η = 3, β = 780 cm, i = x, z; γ = −ν, ν = 1.3, D = 220 cm, D' = 0.3, D'' = D' = 0 at the target location). Effective cross section of the diffusion particle loss σ (i = x, z) and the target mass number A are shown in Fig. 3. The values of the f(ν) function (see Fig. 4) in Eq. (10) show that for high energy heavy projectiles at the Nuclotron (for example, C and Ar nuclei with γ = 7.4) large angle coulomb scattering does not make contribution to σ. When the initial beam emittances (ε(ν)) are known, the corresponding values of T, τ, and T can be obtained from Fig. 3 and Eqs. (9) and (11). Correlation between T, τ, and T is defined by the α(ν) and β(ν) functions, which are plotted in Fig. 5.

The results presented in Figs. 3 and 5 show the substantial contribution of the diffusion process (related to small angle scattering in the target) to the losses (σ(ν)) of the low energy relativistic projectiles. As the energy increases the area of A, where this contribution is negligible, is grown by including larger mass numbers of the target. The maximum values of luminosities averaged over cycle time per one projectile at the first stage of Nuclotron run (Tc = 10 s) are plotted in Fig. 6. As the internal target thickness t [g/cm²] is known (t > t0) and using the presented in Fig. 7 graphical functions of T, the beam lifetime T0 [s] can be estimated. Assuming T0 = Tc in the value of T, the function of the target thickness of Tc can be obtained. As an example, Figs. 6 and 7 also present the curves for deuterons without taking into account the diffusion losses in the transverse phase spaces (σ(ν) = 0).

First physics experiments are being performed at the Nuclotron using the internal target station with thin self-supporting foils of CH₂ (1.6 μm in thickness), Cu (0.55 μm) and Au (1.7 μm) [15]. The dimensions and wall thickness (0.5 mm) of the station are optimized to detect secondary particles by external detectors at a maximum solid angle and with minimum losses. The control

\[ f(ν) = \begin{cases} 2 \times 10^{-2} & \text{for } A = 250 \\ 0.5 \times 10^{-2} & \text{for } A = 250 \\ \end{cases} \]

\[ Z = 1, A = 2 \]

\[ Z = 6, A = 12 \]

\[ Z = 18, A = 40 \]
Fig. 7. The $T_{\text{coll}}$ functions vs target mass number $A_{\text{t}}$ for different incident nuclei at the Nuclotron ($\phi = 1$ AGeV, $\phi = 6$ AGeV). The curves 1 (d, 1 AGeV) and 2 (d, 6 AGeV) present the results when only the $\sigma_{\text{coll}}$ cross section of the deuteron losses is taken into account.

of beam-target interaction is based on the detection of target material radiation by means of the photomultiplier and tandem-microchannel-plate detector. For the above targets and the supposed beam intensities at the first stage of the Nuclotron run (see Refs. [1,16]), the luminosities averaged over cycle time ($L_{c} \left[ \text{cm}^{-2} \text{s}^{-1} \right]$) and the lifetime ($T_{\text{b}} \left[ \text{s} \right]$) of part of the beam interacting with a target are estimated as

\[
L_{c}\{T_{b}\} (d \rightarrow 1 \text{ AGeV}) \approx 2 \times 10^{33} \{3 \times 10^{-7}\} \left( \text{C,CH}_2 \right), \quad 8 \times 10^{31} \{3 \times 10^{-3}\} \left( \text{Cu} \right), \quad 10^{31} \{2 \times 10^{-4}\} \left( \text{Au} \right);
\]

\[
L_{c}\{T_{b}\} (d \rightarrow 6 \text{ AGeV}) \approx 4 \times 10^{33} \{10^{-1}\} \left( \text{C,CH}_2 \right), \quad 10^{32} \{4 \times 10^{-2}\} \left( \text{Cu} \right), \quad 2 \times 10^{32} \{4 \times 10^{-3}\} \left( \text{Au} \right);
\]

\[
L_{c}\{T_{b}\} (C \rightarrow 1 \text{ AGeV}) \approx 2 \times 10^{32} \{3 \times 10^{-2}\} \left( \text{C,CH}_2 \right), \quad 10^{31} \{3 \times 10^{-3}\} \left( \text{Cu} \right), \quad 2 \times 10^{30} \{2 \times 10^{-4}\} \left( \text{Au} \right);
\]

\[
L_{c}\{T_{b}\} (C \rightarrow 6 \text{ AGeV}) \approx 6 \times 10^{32} \{10^{-1}\} \left( \text{C,CH}_2 \right), \quad 10^{32} \{4 \times 10^{-2}\} \left( \text{Cu} \right), \quad 3 \times 10^{31} \{4 \times 10^{-3}\} \left( \text{Au} \right);
\]

\[
L_{c}\{T_{b}\} (Ar \rightarrow 1 \text{ AGeV}) \approx 8 \times 10^{28} \{3 \times 10^{-2}\} \left( \text{C,CH}_2 \right), \quad 6 \times 10^{28} \{4 \times 10^{-3}\} \left( \text{Cu} \right), \quad 10^{28} \{3 \times 10^{-4}\} \left( \text{Au} \right);
\]

\[
L_{c}\{T_{b}\} (Ar \rightarrow 6 \text{ AGeV}) \approx 2 \times 10^{28} \{6 \times 10^{-2}\} \left( \text{C,CH}_2 \right), \quad 6 \times 10^{28} \{4 \times 10^{-3}\} \left( \text{Cu} \right), \quad 2 \times 10^{30} \{5 \times 10^{-3}\} \left( \text{Au} \right).
\]

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